CHAPTER 3

Functions

Section 3A

Function Concepts

- 3-1 Graphing Relationships
- 3-2 Relations and Functions
  - Algebra Lab The Vertical-Line Test
  - Algebra Lab Model Variable Relationships
- 3-3 Writing Functions
- 3-4 Graphing Functions
  - Technology Lab Connect Function Rules, Tables, and Graphs

Section 3B

Applying Functions

- 3-5 Scatter Plots and Trend Lines
  - Technology Lab Interpret Scatter Plots and Trend Lines
- Connecting Algebra to Data Analysis Median-Fit Line
- 3-6 Arithmetic Sequences

Pacing Guide for 45-Minute Classes

Teacher One Stop™ Calendar Planner®

Chapter 3

DAY 1	DAY 2	DAY 3	DAY 4	DAY 5
3-1 Lesson	3-2 Lesson	Algebra Lab Algebra Lab	Algebra Lab 3-3 Lesson	3-3 Lesson
DAY 6	**DAY 7**	**DAY 8**	**DAY 9**	**DAY 10**
3-3 Lesson 3-4 Lesson	3-4 Lesson Technology Lab	Multi-Step Test Prep Ready to Go On?	3-5 Lesson	Technology Lab
DAY 11	**DAY 12**	**DAY 13**		
Connecting Algebra to Data Analysis	3-6 Lesson	Multi-Step Test Prep Ready to Go On?	Chapter Review	Chapter Test

Pacing Guide for 90-Minute Classes

Teacher One Stop™ Calendar Planner®

Chapter 3

DAY 1	DAY 2	DAY 3	DAY 4	DAY 5
3-1 Lesson 3-2 Lesson	Algebra Lab Algebra Lab 3-3 Lesson	3-3 Lesson 3-4 Lesson	3-4 Lesson Technology Lab Multi-Step Test Prep Ready to Go On?	3-5 Lesson Technology Lab
DAY 6	**DAY 7**			
Connecting Algebra to Data Analysis 3-6 Lesson Multi-Step Test Prep Ready to Go On?	Chapter Review Chapter Test			

ONGOING ASSESSMENT and INTERVENTION

	DIAGNOSE	PRESCRIBE
Assess Prior Knowledge	**Before the Chapter**	
	Diagnose readiness for the chapter.	Prescribe intervention.
	Are You Ready? SE	***Are You Ready? Intervention***
Formative Assessment	**Before Every Lesson**	
	Diagnose readiness for the lesson.	Prescribe intervention.
	Warm Up TE	**Reteach** CRB
	During Every Lesson	
	Diagnose understanding of lesson concepts.	Prescribe intervention.
	Check It Out! SE	**Reading Strategies** CRB
	Questioning Strategies TE	***Success for Every Learner***
	Think and Discuss SE	***Lesson Tutorial Videos***
	Write About It SE	
	Journal TE	
	After Every Lesson	
	Diagnose mastery of lesson concepts.	Prescribe intervention.
	Lesson Quiz TE	**Reteach** CRB
	Test Prep SE	**Test Prep Doctor** TE
	ExamView™ Assessment Suite	***Homework Help* Online**
	Before Chapter Testing	
	Diagnose mastery of concepts in chapter.	Prescribe intervention.
	Ready to Go On? SE	***Ready to Go On? Intervention***
	Multi-Step Test Prep SE	**Scaffolding Questions** TE
	Section Quizzes AR	**Reteach** CRB
	ExamView™ Assessment Suite	***Lesson Tutorial Videos***
	Before High Stakes Testing	
	Diagnose mastery of benchmark concepts.	Prescribe intervention.
	College Entrance Exam Practice SE	***College Entrance Exam Practice***
	Standardized Test Prep SE	
Summative Assessment	**After the Chapter Assessment Suite**	
	Check mastery of chapter concepts.	Prescribe intervention.
	Multiple-Choice Tests (Forms A, B, C)	**Reteach** CRB
	Free-Response Tests (Forms A, B, C)	***Lesson Tutorial Videos***
	Performance Assessment AR	
	Cumulative Test AR	
	ExamView™ Assessment Suite	

KEY: **SE** = *Student Edition* **TE** = *Teacher's Edition* **CRB** = *Chapter Resource Book* **AR** = *Assessment Resources* Available online

CHAPTER 3

Lesson Resources

Before the Lesson

Prepare

Teacher One Stop
- Editable lesson plans
- Calendar Planner
- Easy access to all chapter resources

Lesson Transparencies
- Teacher Tools

Teach the Lesson

Introduce

Alternate Openers: Explorations

Lesson Transparencies
- Warm Up
- Problem of the Day

Teach

Lesson Transparencies
- Teaching Transparencies

Know-It Notebook™
- Vocabulary
- Key Concepts

Power Presentations

Lesson Tutorial Videos SPANISH

Interactive Online Edition
- Lesson Activities
- Lesson Tutorial Videos

Lab Activities

Lab Resources Online

Online Interactivities SPANISH

TechKeys

Practice the Lesson

Practice

Chapter Resources
- Practice A, B, C

Practice and Problem Solving Workbook

IDEA Works!® Modified Worksheets and Tests

ExamView™ Assessment Suite

Homework Help Online

Online Interactivities SPANISH

Interactive Online Edition
- Homework Help

Apply

Chapter Resources
- Problem Solving

Practice and Problem Solving Workbook

Interactive Online Edition
- Chapter Project

Project Teacher Support

After the Lesson

Reteach

Chapter Resources
- Reteach
- Reading Strategies ELL

Success for Every Learner

Review

Interactive Answers and Solutions

Solutions Key

Know-It Notebook™
- Big Ideas
- Chapter Review

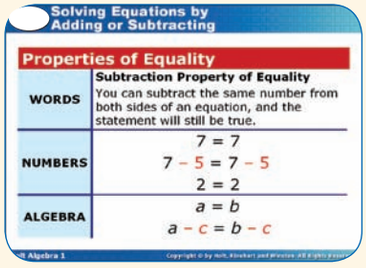
Extend

Chapter Resources
- Challenge

Technology Highlights for the Teacher

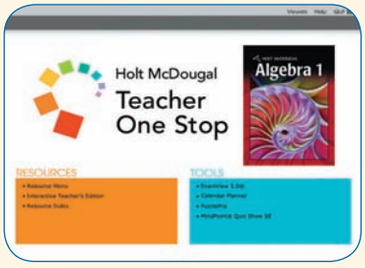
Power Presentations

Dynamic presentations to engage students. Complete PowerPoint® presentations for every lesson in this chapter.



Teacher One Stop SPANISH

Easy access to the chapter resources and assessments. Includes lesson planning software.



Interactive Online Edition

This chapter includes Tutorial Videos, Lesson Activities, Lesson Quizzes, Homework Help, and Chapter Project.

KEY: **SE** = *Student Edition* **TE** = *Teacher's Edition* ELL English Language Learners SPANISH Spanish version available Available online

CHAPTER 3

Reaching All Learners

Teaching tips to help all learners appear throughout the chapter. A few that target specific students are included in the lists below.

All Learners

Lab Activities
Practice and Problem Solving Workbook
Know-It Notebook

Special Needs Students

Practice A ..... CRB
Reteach ..... CRB
Reading Strategies ..... CRB
Are You Ready? ..... SE
Inclusion ..... TE
IDEA Works!® Modified Worksheets and Tests
Ready to Go On? Intervention
Know-It Notebook
Online Interactivities SPANISH
Lesson Tutorial Videos SPANISH

Developing Learners

Practice A ..... CRB
Reteach ..... CRB
Reading Strategies ..... CRB
Are You Ready? ..... SE
Vocabulary Connections ..... SE
Questioning Strategies ..... TE
Ready to Go On? Intervention
Know-It Notebook
Homework Help Online
Online Interactivities SPANISH
Lesson Tutorial Videos SPANISH

On-Level Learners

Practice B ..... CRB
Problem Solving ..... CRB
Vocabulary Connections ..... SE
Questioning Strategies ..... TE
Ready to Go On? Intervention
Know-It Notebook
Homework Help Online
Online Interactivities SPANISH

Advanced Learners

Practice C ..... CRB
Challenge ..... CRB
Challenge Exercises ..... SE
Reading and Writing Math *Extend* ..... TE
Are You Ready? Enrichment
Ready To Go On? Enrichment

English Language Learners

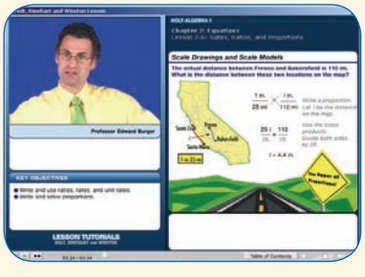
ENGLISH LANGUAGE LEARNERS

Reading Strategies ..... CRB
Are You Ready? Vocabulary ..... SE
Vocabulary Connections ..... SE
Vocabulary Review ..... SE
English Language Learners ..... TE
Success for Every Learner
Know-It Notebook
Multilingual Glossary
Lesson Tutorial Videos SPANISH

Technology Highlights for Reaching All Learners

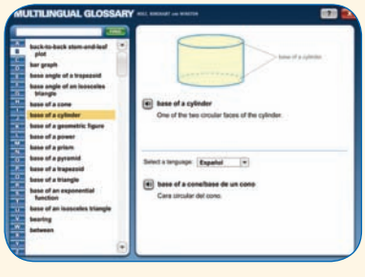
Lesson Tutorial Videos SPANISH

Starring Holt authors Ed Burger and Freddie Renfro! Live tutorials to support every lesson in this chapter.



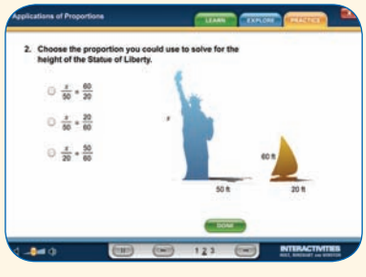
Multilingual Glossary

Searchable glossary includes definitions in English, Spanish, Vietnamese, Chinese, Hmong, Korean, and other languages.



Online Interactivities

Interactive tutorials provide visually engaging alternative opportunities to learn concepts and master skills.



KEY: **SE** = *Student Edition* **TE** = *Teacher's Edition* **CRB** = *Chapter Resource Book* SPANISH Spanish version available Available online

CHAPTER 3

Ongoing Assessment

Assessing Prior Knowledge

Determine whether students have the prerequisite concepts and skills for success in the chapter.

Are You Ready? ..... SE
Warm Up ..... TE

Test Preparation

Provide review and practice for the chapter and standardized tests.

Multi-Step Test Prep ..... SE
Study Guide: Review ..... SE
Test Tackler ..... SE
Standardized Test Prep ..... SE
College Entrance Exam Practice ..... SE
Countdown to Mastery ..... SE
IDEA Works!*® *Modified Worksheets and Tests

Alternative Assessment

Assess students' understanding of the chapter concepts and combined problem-solving skills.

Chapter Project ..... SE
Alternative Assessment ..... TE
Performance Assessment ..... AR
Portfolio Assessment ..... AR

Lesson Assessment

Provide formative assessment for each lesson of the chapter.

Questioning Strategies ..... TE
Think and Discuss ..... SE
Check It Out! Exercises ..... SE
Write About It ..... SE
Journal ..... TE
Lesson Quiz ..... TE
Alternative Assessment ..... TE
IDEA Works!*® *Modified Worksheets and Tests

Weekly Assessment

Provide formative assessment for each section of the chapter.

Multi-Step Test Prep ..... SE
Ready to Go On? ..... SE
Section Quizzes ..... AR
ExamView*™ *Assessment Suite

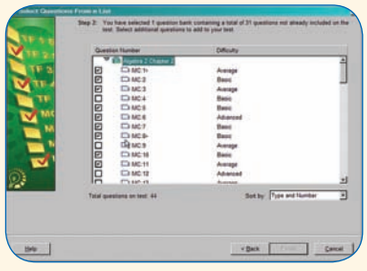
Chapter Assessment

Provide summative assessment of the chapter mastery.

Chapter Test ..... SE
Chapter Test (Levels A, B, C) ..... AR
• Multiple Choice • Free Response
Cumulative Test ..... AR
ExamView*™ *Assessment Suite
IDEA Works!*® *Modified Worksheets and Tests

Technology Highlights for Assessment

ExamView™ Assessment Suite
Use the chapter problem banks to create assessments and worksheets to print out or deliver online. Includes dynamic problems.



KEY: **SE** = *Student Edition* **TE** = *Teacher's Edition* **AR** = *Assessment Resources* SPANISH Spanish version available Available online

CHAPTER 3

Formal Assessment

Three levels (A, B, C) of multiple-choice and free-response chapter tests, along with a performance assessment, are available in the *Assessment Resources.*

MULTIPLE CHOICE

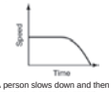
A Chapter Test

C Chapter Test

B Chapter Test

Select the best answer.

1. Which situation could be represented by the graph below?



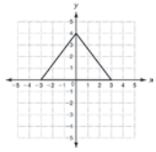
A A person slows down and then travels at a constant speed.
B A person travels at a constant speed and then slows down.
C A person travels at a constant speed and then speeds up.
D A person speeds up and then travels at a constant speed.

2. Which situation would NOT be represented by a graph with distinct points?
F Cost of buying 1, 2, or 3 packs of trading cards
G Number of visitors to a movie theater per day for one week
H Distance traveled by a rollercoaster car during 20 seconds
J Amount of money earned based on the number of magazines sold

3. Which relation is represented by the mapping diagram below?

A {(3, 2), (5, 2), (7, 4)}
B {(2, 3), (4, 7)}
C {(5, 2), (7, 4)}
D {(2, 3), (2, 5), (4, 7)}

4. What is the domain of the relation below?



F $0 \le x \le 3$
G $0 \le x \le 4$
H $-3 \le x \le 3$
J $-3 \le x \le 4$

5. What is the range of the relation below?

x	0	1	2	3
y	1	2	4	8

A {1, 2}
B {0, 1, 2, 3}
C {1, 2, 4, 8}
D {0, 1, 2, 3, 4, 8}

6. Which of the following relations is NOT a function?
F {(1, 5), (3, −1), (7, 9)}
G {(1, 5), (2, 5)}
H {(1, 1), (2, 2), (3, 3)}
J {(5, 1), (5, 2)}

7. Which equation shows the relationship between x and y in {(1, −3), (2, −1), (3, 1)}?
A $y = 2x - 5$
B $y = x - 4$
C $y = 2x$
D $y = x - 2$

B Chapter Test *(continued)*

8. Which function could represent the following situation: "An internet cafe charges $0.20 per minute."
F $f(m) = m + 0.20$
G $f(m) = \frac{m}{20}$
H $f(m) = 0.20m$
J $f(m) = 20m$

9. An architect must convert 216, 183, and 129 yards to feet. Which function rule describes the situation?
A $f(y) = \frac{y}{3}$
B $f(y) = 3y$
C $f(y) = \frac{3}{y}$
D $f(y) = y - 3$

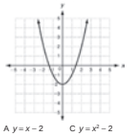
10. Evaluate the function $f(x) = 3x^2 - 5$ when $x = -11$.
F −358
G −38
H 38
J 358

11. Which is the dependent variable in the following situation?
"Milk sells for $3.39 per gallon."
A number of gallons
B total cost
C expiration date
D time of purchase

12. Which function is graphed for the domain {−2, 0, 2, 4}?

F $y = x - 1$
G $y = |x| - 1$
H $y = x^2 - 1$
J $y = |x - 1|$

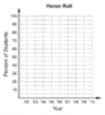
13. Which function is graphed below?



A $y = x - 2$
B $y = 2x$
C $y = x^2 - 2$
D $y = 2x + 1$

14. The table shows the percent of students on the honor roll for four years. Draw a scatter plot and trend line.

Year	Honor Roll
'02	35%
'03	42%
'04	38%
'05	48%



Which is the best prediction for the percent of students on the honor roll in 2009?
F 35%
G 40%
H 55%
J 70%

15. Find the next three terms of the arithmetic sequence 5, 11, 17, 23, ...
A 29, 34, 38
B 28, 33, 38
C 25, 31, 37
D 29, 35, 41

16. What is the 18th term of the arithmetic sequence 2, −2, −6, −10,...?
F −72
G −70
H −68
J −66

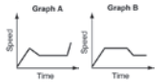
FREE RESPONSE

A Chapter Test

C Chapter Test

B Chapter Test

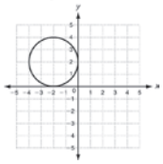
1. A runner in a race ran quickly for the first few minutes, slowed down some and ran a steady pace for most of the race, and then ran as fast as he could at the very end. Choose the graph that best represents this situation.



2. Dog food is sold in 7 pound bags. Sketch a graph to show the weight of dog food purchased by a customer who buys 0, 1, 2, 3, or 4 bags. Tell whether the graph is continuous or discrete.

3. Express the relation {(−2, 3), (2, 4), (−3, 4)}, as a mapping diagram.

4. Give the range of the relation.



5. Give the domain of the relation.

x	−1	−0.5	0	0.5
y	2	2	5	6

6. Tell whether the relation is a function. Explain.
{(4, −1), (3, −2), (2, 1), (1, −2)}

7. Determine a relationship between the x- and y-values. Write an equation.

x	1	2	3	4	5
y	2	5	8	11	14

B Chapter Test *(continued)*

Write a rule in function notation for each situation.

8. A car can travel 32.5 miles per gallon of gasoline.

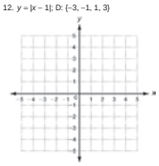
9. A lawyer will be paid $\frac{1}{3}$ of the amount awarded in a lawsuit.

10. Evaluate the function $f(x) = (2x)^2 - 1$ when $x = 2$ and when $x = -1$.

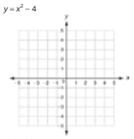
11. Identify the independent and dependent variables.
A certain movie earns $1200 for each screen it is shown on.

Graph each function.

12. $y = |x - 1|$; D: {−3, −1, 1, 3}

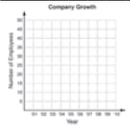


13. $y = x^2 - 4$



14. The table shows the number of employees in a company over five years. Draw a scatter plot and trend line.

Year	'01	'02	'03	'04	'05
Employees	15	20	23	30	34



Based on the trend line, predict how many employees the company will have in 2008.

15. Find the next three terms of the arithmetic sequence 8, 14, 20, 26, ...

16. What is the 57th term of the arithmetic sequence 11, 8, 5, 2,...?

PERFORMANCE ASSESSMENT

Chapter Test

Text Messaging

Purpose
This performance task assesses the student's ability to find the domain and range of relations, and to identify functions.

Time
25–40 minutes

Grouping
Individuals

Preparation Hints
Review the definitions for *relation*, *domain*, *range*, and *function*. Talk about the different ways to represent relations: ordered pairs, tables, graphs, and mapping diagrams.

Overview
Typing a text message on a standard cell phone is a real-world application of relations and functions. When you type a text message, the relation between the letter you want and the numbered key you press *is* a function. However, when the phone deciphers the numbered keys as letters to display on the screen, the relation is *not* a function.

Introduce the Task
Ask how many students have cell phones, and how many have used them to send a text message. Discuss different strategies, techniques, and technology for physically typing in a text message. Phones vary widely; some use predictive text messaging, some require the user to press the keys different numbers of times for each letter, and some have extended keypads with each letter corresponding to one key.

Performance Indicators
_____ Expresses a relation as a table.
_____ Finds the domain and range of relations.
_____ Distinguishes relations and functions.
_____ Finds outputs given specific inputs by using a relation's table.
_____ Makes a prediction for a real-life application.

Scoring Rubric
Level 4: Student solves all problems correctly and gives explanations.
Level 3: Student solves all problems correctly but gives no explanations.
Level 2: Student solves some problems but gives no explanations.
Level 1: Student is not able to solve any of the problems.

Chapter Test *(continued)*

Text Messaging

The keypad on Suki's cell phone is shown at right. As you can see, there is a relationship between the letters and numbers.

1	ABC 2	DEF 3
GHI 4	JKL 5	MNO 6
PQRS 7	TUV 8	WXYZ 9
*	space 0	#

When Suki writes a text message, she thinks of the letter in the word she is typing and presses the correct numbered key.

1. Complete this table to show the relation from letter to number.

Letter	Number	Letter	Number	Letter	Number	Letter	Number
A	2	H					
B	2	I					
C	2						
D							
E							
F							
G							

2. What is the domain of this relation?
3. What is the range of this relation?
4. Is the relation from letter to number a function? Explain.
5. If Suki writes the word "Hello," what sequence of numbers does she press?

After Suki presses the numbered keys, the microchip within her cell phone must decide what letters to show on the screen. Consider the relation from *number to letter*.

6. What is the domain of this relation?
7. What is the range of this relation?
8. Is the relation from number to letter a function? Explain.
9. List the possible three-letter combinations the phone could show on the screen after Suki presses 3 6 4.
10. Suki's phone uses *predictive text messaging*, which means that it "guesses" what word she meant to type. Which three-letter word in problem 9 do you think the

CHAPTER 3

MATHEMATICAL PRACTICES **Reason abstractly and quantitatively.**

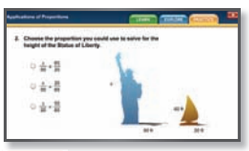
SECTION 3A
Function Concepts

MULTI-STEP TEST PREP Students use a graph to write an equation of a function that models the volume of water in a pool.

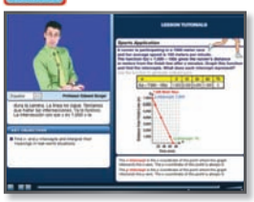
SECTION 3B
Applying Functions

MULTI-STEP TEST PREP Students create a scatter plot and trend line to model a real-world travel situation.

Interactivities Online ▶



Video Lesson Tutorials Online



Lesson Tutorial Videos are available for EVERY example.

CHAPTER 3

Functions

COMMON CORE

3A Function Concepts

3-1	Graphing Relationships	F.IF.4*
3-2	Relations and Functions	F.IF.1
Lab	The Vertical-Line Test	F.IF.1
Lab	Model Variable Relationships	
3-3	Writing Functions	F.IF.2
3-4	Graphing Functions	F.IF.5*
Lab	Connect Function Rules, Tables, and Graphs	F.IF.1

3B Applying Functions

3-5	Scatter Plots and Trend Lines	S.ID.6
Lab	Interpret Scatter Plots and Trend Lines	S.ID.6
3-6	Arithmetic Sequences	F.IF.3

Chapter Focus

- Use tables, diagrams, graphs, and equations to describe functions.
- Translate among representations of functions.
- Use functions to represent, analyze, and solve problems.

Is That *Your* Foot?

Criminologists use measurements, such as the size of footprints, and functions to help them identify criminals.

Learn It Online
Chapter Project Online

© David McGlynn/FlatEarth/Getty Images

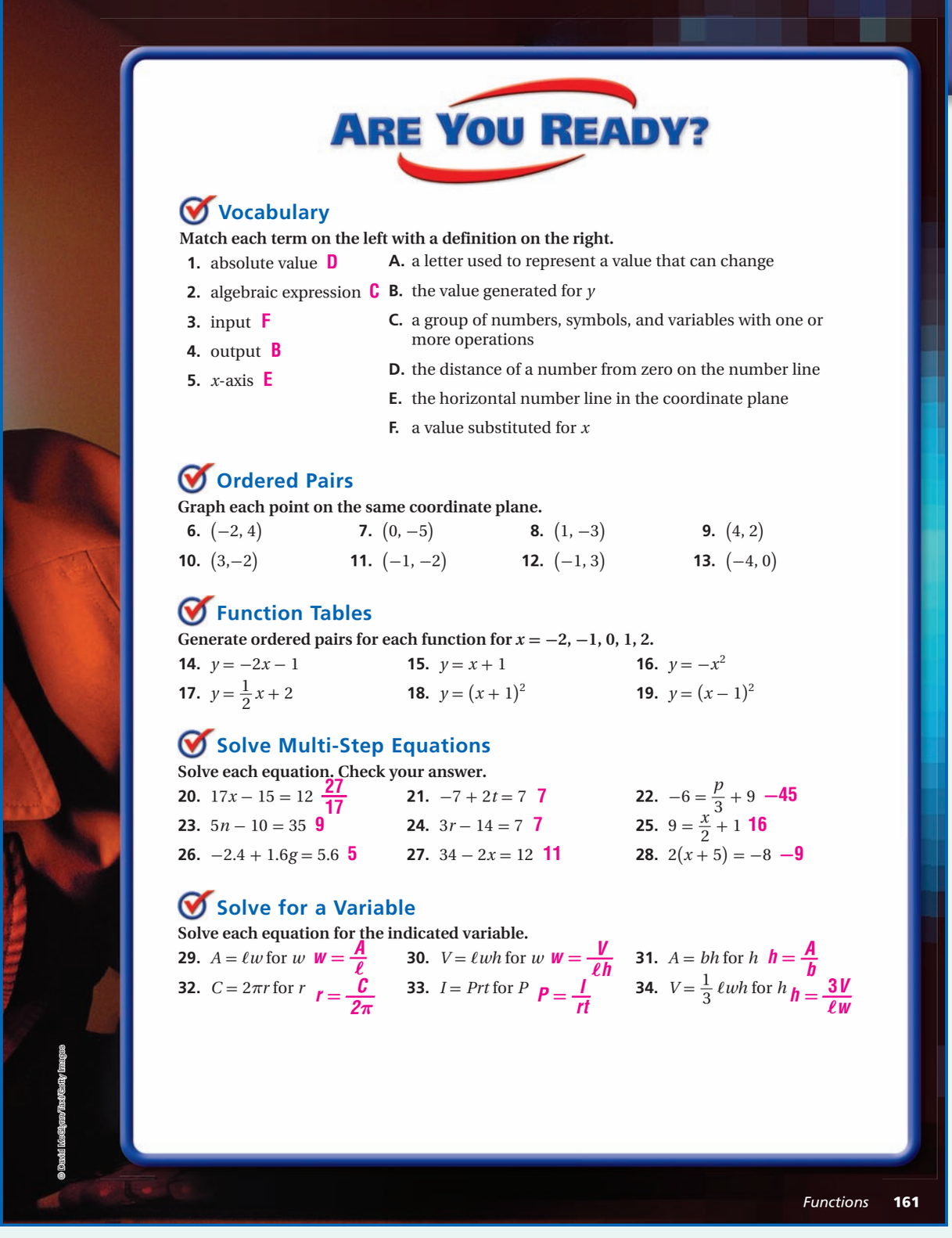
Chapter Project

A project for this chapter is available online.

Project Resources

All project resources for teachers and students are provided online.

Learn It Online
Chapter Project Online



ARE YOU READY?

Vocabulary

Match each term on the left with a definition on the right.

1. absolute value **D**
2. algebraic expression **C**
3. input **F**
4. output **B**
5. x-axis **E**

A. a letter used to represent a value that can change
B. the value generated for y
C. a group of numbers, symbols, and variables with one or more operations
D. the distance of a number from zero on the number line
E. the horizontal number line in the coordinate plane
F. a value substituted for x

Ordered Pairs

Graph each point on the same coordinate plane.

6. $(-2, 4)$
7. $(0, -5)$
8. $(1, -3)$
9. $(4, 2)$
10. $(3, -2)$
11. $(-1, -2)$
12. $(-1, 3)$
13. $(-4, 0)$

Function Tables

Generate ordered pairs for each function for $x = -2, -1, 0, 1, 2$.

14. $y = -2x - 1$
15. $y = x + 1$
16. $y = -x^2$
17. $y = \frac{1}{2}x + 2$
18. $y = (x + 1)^2$
19. $y = (x - 1)^2$

Solve Multi-Step Equations

Solve each equation. Check your answer.

20. $17x - 15 = 12$ **$\frac{27}{17}$**
21. $-7 + 2t = 7$ **7**
22. $-6 = \frac{p}{3} + 9$ **-45**
23. $5n - 10 = 35$ **9**
24. $3r - 14 = 7$ **7**
25. $9 = \frac{x}{2} + 1$ **16**
26. $-2.4 + 1.6g = 5.6$ **5**
27. $34 - 2x = 12$ **11**
28. $2(x + 5) = -8$ **-9**

Solve for a Variable

Solve each equation for the indicated variable.

29. $A = \ell w$ for w **$w = \frac{A}{\ell}$**
30. $V = \ell wh$ for w **$w = \frac{V}{\ell h}$**
31. $A = bh$ for h **$h = \frac{A}{b}$**
32. $C = 2\pi r$ for r **$r = \frac{C}{2\pi}$**
33. $I = Prt$ for P **$P = \frac{I}{rt}$**
34. $V = \frac{1}{3}\ell wh$ for h **$h = \frac{3V}{\ell w}$**

© David McGlynn/Taxi/Getty Images

CHAPTER 3

ARE YOU READY?

Organizer

Objective: Assess students' understanding of prerequisite skills.

Assessing Prior Knowledge

INTERVENTION

Diagnose and Prescribe

Use this page to determine whether intervention is necessary or whether enrichment is appropriate.

Resources

***Are You Ready? Intervention and Enrichment* Worksheets**

***Are You Ready?* Online**

Answers

6–19. See Additional Answers.

ARE YOU READY?

Diagnose and Prescribe

NO INTERVENE

	Are You Ready? Intervention, Chapter 3		
Prerequisite Skill	**Worksheets**	**Online**	
Ordered Pairs	Skill 79	Activity 79	Diagnose and Prescribe Online
Function Tables	Skill 78	Activity 78	
Solve Multi-Step Equations	Skill 69	Activity 69	
Solve for a Variable	Skill 72	Activity 72	

YES ENRICH

Are You Ready? Enrichment, Chapter 3
Worksheets
Online

Organizer

Objective: Help students organize the new concepts they will learn in this chapter.

Online Edition
Multilingual Glossary

Resources

Learn It Online
Multilingual Glossary

Possible answers to *Vocabulary Connections*

1. Relying on someone or something else; the value of a dependent variable relies on the value of another variable.
2. Function notation is a method of writing special types of relations.
3. The two data sets have no relationship.
4. Something about them is the same. When you subtract two consecutive numbers in the list, the answer is the same as when you subtract two other consecutive numbers.

CHAPTER 3 Study Guide: Preview

Where You've Been

Previously, you

- were introduced to functions when you generated and graphed ordered pairs.
- stated rules for relationships among values.
- represented and interpreted data using bar graphs and circle graphs.

In This Chapter

You will study

- relationships between variables and determine whether a relation is a function.
- relationships in function notation.
- how trend lines on scatter plots can help you make predictions.

Where You're Going

You can use the skills in this chapter

- to find values of a function from a graph.
- to analyze data and make predictions in other courses, such as Chemistry.
- to calculate total earnings for a certain hourly rate.

Key Vocabulary/Vocabulario

arithmetic sequence	sucesión aritmética
common difference	diferencia común
correlation	correlación
dependent variable	variable dependiente
domain	dominio
function	función
function notation	notación de función
independent variable	variable independiente
no correlation	sin correlación
range	rango
relation	relación
scatter plot	diagrama de dispersión
sequence	sucesión

Vocabulary Connections

To become familiar with some of the vocabulary terms in the chapter, consider the following. You may refer to the chapter, the glossary, or a dictionary if you like.

1. What does the word *dependent* mean? What do you think is true about the value of a **dependent variable**?
2. A *function* is a special type of relation and *notation* is a method of writing. What do you suppose is meant by **function notation**?
3. The word *correlation* means "relationship." What might it mean if two sets of data have **no correlation**?
4. What does it mean when someone says that two people have something in *common*? If *difference* is the answer to a subtraction problem, what might it mean for a list of numbers to have a **common difference**?

MATHEMATICAL PRACTICES

The Common Core Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. Opportunities to develop these practices are integrated throughout this program. Some examples are provided below.

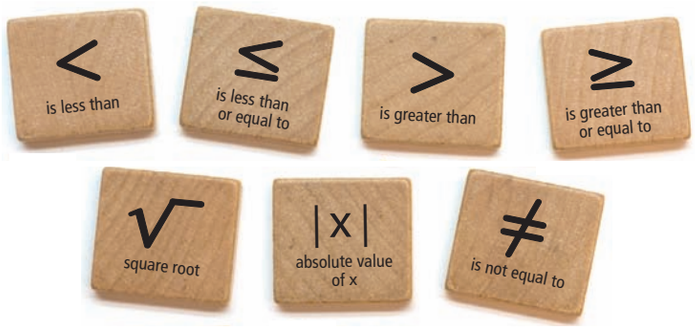
1. **Make sense of problems and persevere in solving them.** Pages 168, 174–175, 183–185, 189, 190–192, 194, 200–203, 209–211, 212
2. **Reason abstractly and quantitatively.** Pages 184–185, 192, 200–203
3. **Construct viable arguments and critique the reasoning of others.** Pages 166, 168, 172, 174–175, 182, 184, 190, 191–192, 199, 202, 208, 209–210
4. **Model with mathematics.** Pages 164–166, 167–169, 180–182
5. **Use appropriate tools strategically.** Pages 168, 177, 187, 193, 204
6. **Attend to precision.** Pages 184, 189, 192, 202
7. **Look for and make use of structure.** Pages 173–174, 183–184, 190–192, 209–211
8. **Look for and express regularity in repeated reasoning.** Pages 177, 178, 206–208

162 Chapter 3

Reading Strategy: Read and Interpret Math Symbols

It is essential that as you read through each lesson of the textbook, you can interpret mathematical symbols.

Common Math Symbols



You must be able to translate symbols into words . . .

Using Symbols	Using Words
$3\left(\frac{x}{12}\right) - 1 = 21$	Three times the quotient of x and 12, minus 1 **equals** 21.
$25x + 6 \geq 17$	Twenty-five times x plus 6 **is greater than or equal** to 17.
$\lvert x \rvert > 14$	The **absolute value** of x **is greater than** 14.
$\sqrt{60 + x} \leq 40$	The **square root** of the sum of 60 and x **is less than or equal** to 40.

. . . and words into symbols.

Using Words	Using Symbols
The height of the shed **is at least** 9 feet.	$h \geq 9$ ft
The distance **is at most** one tenth of a mile.	$d \leq 0.1$ mi
The silo contains **more than** 600 cubic feet of corn.	$c > 600$ ft^3

Try This

Translate the symbols into words.

1. $x \leq \sqrt{10}$
2. $\lvert x \rvert + 2 > 45$
3. $-5 \leq x < 8$
4. $-6 - \frac{1}{5}x = -32$

Translate the words into symbols.

5. There are less than 15 seconds remaining.
6. The tax rate is 8.25 percent of the cost.
7. Ann counted over 100 pennies.
8. Joe can spend at least \$22 but no more than \$30.

Standards for Mathematical Content High School

		LESSON
INTERPRETING FUNCTIONS Understand the concept of a function and use function notation		
CC.9-12.F.IF.1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element in its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.	3-2, Lab 3-2, 3-3, 3-4, TLab 3-4
CC.9-12.F.IF.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.	3-3, 3-4
CC.9-12.F.IF.3	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.	3-6
INTERPRETING FUNCTIONS Interpret functions that arise in applications in terms of the context		
CC.9-12.F.IF.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.*	3-1
CC.9-12.F.IF.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.*	3-2, 3-3, 3-4
INTERPRET CATEGORICAL AND QUANTITATIVE DATA Summarize, represent, and interpret data on two categorical and quantitative variables		
CC.9-12.S.ID.6	Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.	3-5

Organizer

Objective: Help students apply strategies to understand and retain key concepts.

Online Edition

ENGLISH LANGUAGE LEARNERS

Reading Strategy: Read and Interpret Math Symbols

Discuss Mathematics is considered an "international language" because the same symbols are used all around the world.

Extend As students work through this chapter point out how symbols make writing some answers (such as domain and range) easier than if they had used only words.

Answers to *Try This*

Possible answers:

1. x is less than or equal to the square root of 10.
2. The sum of the absolute value of x and 2 is greater than 45.
3. x is greater than or equal to negative 5 and is less than 8.
4. Negative 6 minus $\frac{1}{5}$ times x is equal to negative 32.
5. $t < 15$
6. $r = 0.0825$
7. $a > 100$
8. $22 \leq s \leq 30$

Reading Connection

***The Man Who Counted* by Malba Tahan**

This is a book of classic puzzles written in an exotic style reminiscent of *1001 Arabian Nights*. A Persian mathematician travels the countryside. At each stop, a situation arises that requires an elegant solution to a math problem.

Activity After students study arithmetic sequences, have them read Chapter 16, *The Game Plan*. Ask them to explain the method used to allot grains of com to the chess-board squares and to compare that method with an arithmetic sequence.

SECTION 3A

Function Concepts

One-Minute Section Planner

Lesson	Lab Resources	Materials
Lesson 3-1 Graphing Relationships • Match simple graphs with situations. • Graph a relationship. ☐ SAT-10 ☑ NAEP ☐ ACT ☑ SAT ☐ SAT Subject Tests	***Technology Lab Activities*** Technology Lab	
Lesson 3-2 Relations and Functions • Identify functions. • Find the domain and range of relations and functions. ☑ SAT-10 ☑ NAEP ☑ ACT ☑ SAT ☑ SAT Subject Tests		
Algebra Lab The Vertical-Line Test • Determine if a relation is a function. ☑ SAT-10 ☑ NAEP ☑ ACT ☑ SAT ☐ SAT Subject Tests	***Algebra Lab Activities*** Lab Recording Sheet	**Optional** graphing calculator
Algebra Lab Model Variable Relationships • Use models to represent algebraic relationships. ☐ SAT-10 ☐ NAEP ☐ ACT ☐ SAT ☐ SAT Subject Tests	***Algebra Lab Activities*** Lab Recording Sheet	**Optional** square tiles
Lesson 3-3 Writing Functions • Identify independent and dependent variables. • Write an equation in function notation and evaluate a function for given input values. ☑ SAT-10 ☑ NAEP ☑ ACT ☑ SAT ☑ SAT Subject Tests	***Algebra Lab Activities*** Algebra Lab	
Lesson 3-4 Graphing Functions • Graph functions given a limited domain. • Graph functions given a domain of all real numbers. ☐ SAT-10 ☑ NAEP ☑ ACT ☑ SAT ☑ SAT Subject Tests		**Optional** graphing calculator, scale, pennies
Technology Lab Connect Function Rules, Tables, and Graphs • Use a graphing calculator to make the connections among function rules, tables, and graphs. ☑ SAT-10 ☑ NAEP ☑ ACT ☑ SAT ☐ SAT Subject Tests	***Technology Lab Activities*** Lab Recording Sheet	**Required** graphing calculator

Note: If NAEP is checked, the content is tested on either the Grade 8 or Grade 12 NAEP assessment.

MK = *Manipulatives Kit*

Selected Answers in Student Edition

The answers to the following exercises are included in the Selected Answers section at the back of the Student Edition textbook.

Lesson 1: 1, 3, 5, 11, 13, 19, 23

Lesson 2: 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29a, 29b, 29c, 33, 35a, 35b, 35c, 37

Lesson 3: 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 33, 35, 37

Lesson 4: 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 29, 31, 33, 35, 37, 39, 41, 43, 45, 47, 55a, 55b, 55c, 59, 61, 63

All answers for the Check it Out! exercises in each lesson and for the Chapter Study Guide: Review are included in the Student Edition.

Math Background

COMMON CORE PROFESSIONAL DEVELOPMENT

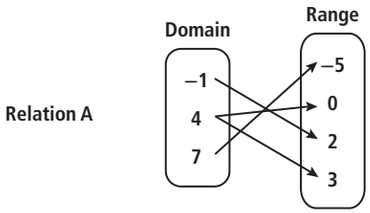
RELATIONS AND FUNCTIONS

Lesson 3-2

Many students have the mistaken belief that relations and functions must be described by equations. Presenting relations and functions as sets of ordered pairs right from the start helps avoid this misconception.

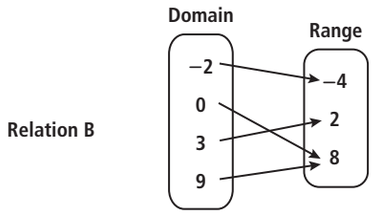
A *relation* is simply a set of ordered pairs. For a given relation, the set of all first values (or *x*-values) is the *domain.* The set of all second values (or *y*-values) is the *range.*

The ordered pairs in a relation may be given as a set. For example, the relation described by the set {(−1, 2), (4, 0), (4, 3), (7, −5)} consists of four ordered pairs. The ordered pairs may also be presented in a table, graph, or a mapping diagram. This mapping diagram below describes the same relation as the set of ordered pairs.

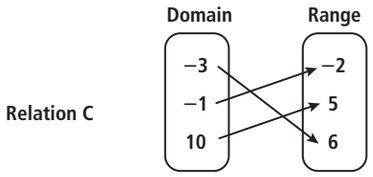


A relation can be described by the way it matches domain and range values. Relation A is *one-to-many.* This means there is at least one domain value (in this case, 4) that is paired with more than one range value (0 and 3).

A relation is *many-to-one* if more than one domain value is paired with a single range value. Relation B, shown below, is many-to-one because the domain values 0 and 9 are both paired with the range value 8.



A relation is *one-to-one* if each domain value is paired with a unique range value and each range value is paired with a unique domain value. Relation C is one-to-one.



FUNCTIONS

Lessons 3-2, 3-3

A *function* is a relation in which each domain value is paired with exactly one range value. Thus, every function is a relation, but not every relation is a function. In the above examples, relations B and C are functions. Relation A is not a function because the domain value 4 is paired with more than one range value.

The definition of *function* may seem somewhat arbitrary to students. It is helpful to explicitly point out the important role that functions play in real-world situations. For example, a meteorologist might study the relation in which each day of the year is paired with the daily high temperature in Los Angeles. It would not make sense if one domain value (a day of the year) were paired with more than one range value (a high temperature). This relation is a function, as are most relations that describe real-world situations. This example illustrates that a function does not have to be defined with an equation or formula; the function above is not described by any formula that we understand.

The real-world example also illustrates why the first value of a function is the *independent variable* and the second value is the *dependent variable.* The second value (e.g., temperature) *depends* on the first value (e.g., day of the year).

Functions may be one-to-one or many-to-one. The function that gives the daily high temperature in Los Angeles is likely to be many-to-one, as there may be several days that are paired with the same temperature. However, by definition a function cannot be one-to-many.

3-1 Organizer

Pacing: Traditional 1 day
Block $\frac{1}{2}$ day

Objectives: Match simple graphs with situations.

Graph a relationship.

Technology Lab
In *Technology Lab Activities*

Online Edition
Tutorial Videos

Countdown

Power Presentations with PowerPoint®

Warm Up

State whether each word or phrase represents an amount that is increasing, decreasing, or constant.

1. stays the same constant
2. rises increasing
3. drops decreasing
4. slows down decreasing

Also available online

Math Humor

Q: What do a math teacher and an English teacher have in common?

A: They can both make a "pair-a-graph."

COMMON CORE

CC.9-12.F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.*

CC.9-12.N.Q.2 Define appropriate quantities for the purpose of descriptive modeling.*

Learn It Online
State Resources Online

COMMON CORE

3-1

Graphing Relationships

CC.9-12.F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs…, and sketch graphs showing key features given a verbal description of the relationship.* *Also* **CC.9-12.N.Q.2***

Objectives
Match simple graphs with situations.

Graph a relationship.

Vocabulary
continuous graph
discrete graph

Who uses this?

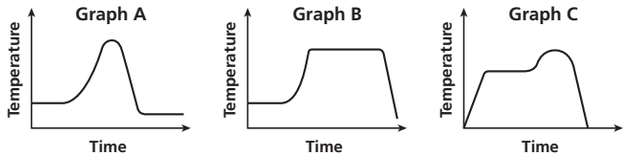
Cardiologists can use graphs to analyze their patients' heartbeats. (See Example 2.)

Graphs can be used to illustrate many different situations. For example, trends shown on a cardiograph can help a doctor see how the patient's heart is functioning.

To relate a graph to a given situation, use key words in the description.

EXAMPLE 1 Relating Graphs to Situations

The air temperature was constant for several hours at the beginning of the day and then rose steadily for several hours. It stayed the same temperature for most of the day before dropping sharply at sundown. Choose the graph that best represents this situation.



Step 1 Read the graphs from left to right to show time passing.

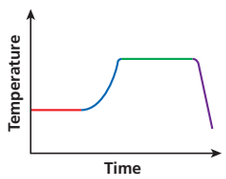
Step 2 List key words in order and decide which graph shows them.

Key Words	Segment Description	Graphs
Was constant	Horizontal	Graphs A and B
Rose steadily	Slanting upward	Graphs A and B
Stayed the same	Horizontal	Graph B
Dropped sharply	Slanting downward	Graph B

Step 3 Pick the graph that shows all the key phrases in order.

horizontal, slanting upward, horizontal, slanting downward

The correct graph is B.

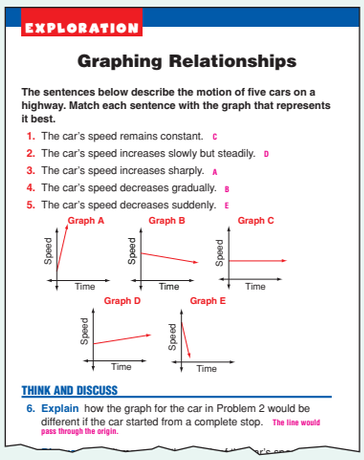


CHECK IT OUT!

1. The air temperature increased steadily for several hours and then remained constant. At the end of the day, the temperature increased slightly again before dropping sharply. Choose the graph above that best represents this situation. C

© Ocean/CORBIS

1 Introduce



EXPLORATION

Graphing Relationships

The sentences below describe the motion of five cars on a highway. Match each sentence with the graph that represents it best.

1. The car's speed remains constant. C
2. The car's speed increases slowly but steadily. D
3. The car's speed increases sharply. A
4. The car's speed decreases gradually. B
5. The car's speed decreases suddenly. E

Graph A, Graph B, Graph C, Graph D, Graph E
Speed
Time

THINK AND DISCUSS

6. **Explain** how the graph for the car in Problem 2 would be different if the car started from a complete stop. The line would pass through the origin.

Motivate

Have students state what would happen to the height of an ice cube in the following situations.

It is placed in a refrigerator. decrease slowly

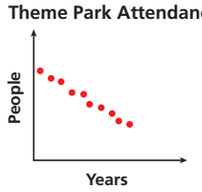
It is placed in a hot oven. decrease quickly

It is placed in a freezer. stay the same

Explorations and answers are provided in your online resources.

As seen in Example 1, some graphs are connected lines or curves called **continuous graphs**. Some graphs are only distinct points. These are called **discrete graphs**.

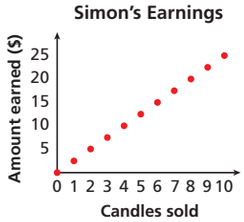
The graph on theme-park attendance is an example of a discrete graph. It consists of distinct points because each year is distinct and people are counted in whole numbers only. The values between the whole numbers are not included, since they have no meaning for the situation.



EXAMPLE 2 Sketching Graphs for Situations

Sketch a graph for each situation. Tell whether the graph is continuous or discrete.

A **Simon is selling candles to raise money for the school dance. For each candle he sells, the school will get $2.50. He has 10 candles that he can sell.**



The amount earned (y-axis) increases by $2.50 for each candle Simon sells (x-axis).

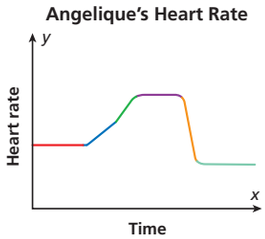
Since Simon can only sell whole numbers of candles, the graph is 11 distinct points.

The graph is discrete.

B **Angelique's heart rate is being monitored while she exercises on a treadmill. While walking, her heart rate remains the same. As she increases her pace, her heart rate rises at a steady rate. When she begins to run, her heart rate increases more rapidly and then remains high while she runs. As she decreases her pace, her heart rate slows down and returns to her normal rate.**

As time passes during her workout (moving left to right along the *x*-axis), her heart rate (*y*-axis) does the following:

- remains the same,
- rises at a steady rate,
- increases more rapidly (steeper than previous segment),
- remains high,
- slows down,
- and then returns to her normal rate.



The graph is continuous.

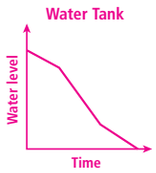
CHECK IT OUT! **Sketch a graph for each situation. Tell whether the graph is continuous or discrete.**

2a. Jamie is taking an 8-week keyboarding class. At the end of each week, she takes a test to find the number of words she can type per minute. She improves each week.

2b. Henry begins to drain a water tank by opening a valve. Then he opens another valve. Then he closes the first valve. He leaves the second valve open until the tank is empty.

2a. discrete;

b. continuous



2 Teach

Guided Instruction

Explain that all graphs should be read from left to right. Encourage students to write down key phrases from a situation in the order they appear to help with matching situations to graphs. Help students identify which label to put on which axis by having them ask themselves which variable depends on the other variable. For instance, "Does time depend on distance, or does distance depend on time?" The dependent variable (distance) goes on the *y*-axis.

Reaching All Learners

Through Cooperative Learning

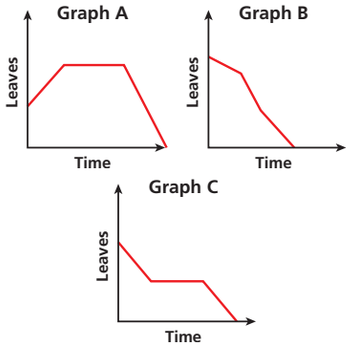
Put students in groups of three or four. Instruct each group to write a situation on one piece of paper and draw a corresponding graph on a separate piece of paper. Collect all graphs and shuffle them; then collect all situations and shuffle. Randomly label graphs A, B, C, . . . , and situations 1, 2, 3, ... Display all graphs and situations on the wall. Have groups match each graph with the correct description.

Power Presentations with PowerPoint®

Additional Examples

Example 1

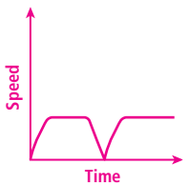
Each day several leaves fall from a tree. One day a gust of wind blows off many leaves. Eventually, there are no more leaves on the tree. Choose the graph that best represents this situation. B



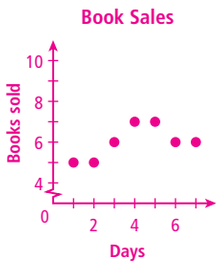
Example 2

Sketch a graph for each situation. Tell whether the graph is continuous or discrete.

A. A truck driver enters a street, drives at a constant speed, stops at a light, and then continues. continuous



B. A small bookstore sold between 5 and 8 books each day for 7 days. discrete; possible answer:



Also available online

Questioning Strategies

EXAMPLES 1–2

- What are some phrases used to describe a graph that slants upward? downward? remains constant?
- How do you know when to use discrete points instead of connected segments or curves?

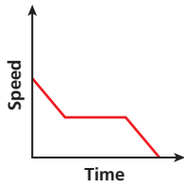
Inclusion Some students may interpret a horizontal line to mean that the *x*-value is constant. Label some points on the line to show that it is the *y*-value that is constant.

Power Presentations with PowerPoint®

Additional Examples

Example 3

Write a possible situation for the given graph.



Possible answer: A car approaching traffic slows down, drives at a constant speed, and then slows down until coming to a complete stop.

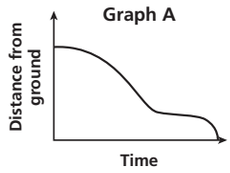
Also available online

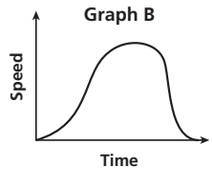
Questioning Strategies

EXAMPLE 3

- What part of the graph shows "increased steadily"? "remained unchanged"? "decreased steadily"?

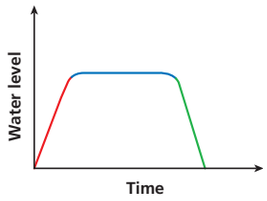
When sketching or interpreting a graph, pay close attention to the labels on each axis. Both graphs below show a relationship about a child going down a slide. **Graph A** represents the child's *distance from the ground* over time. **Graph B** represents the child's *speed* over time.





EXAMPLE 3 Writing Situations for Graphs

Write a possible situation for the given graph.



Step 1 Identify labels.
x-axis: time *y*-axis: water level

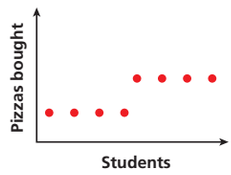
Step 2 Analyze sections.
Over time, the water level
- increases steadily,
- remains unchanged,
- and then decreases steadily.

Possible Situation: A watering can is filled with water. It sits for a while until some flowers are planted. The water is then emptied on top of the planted flowers.

CHECK IT OUT!

3. Write a possible situation for the given graph.

Possible answer: When the number of students reaches a certain point, the number of pizzas bought increases.

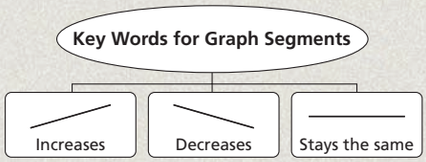


MATHEMATICAL PRACTICES

THINK AND DISCUSS

1. Should a graph of age related to height be a continuous graph or a discrete graph? Explain.
2. Give an example of a situation that, when graphed, would include a horizontal segment.
3. **GET ORGANIZED** Copy and complete the graphic organizer. Write an example of key words that suggest the given segments on a graph. One example for each segment is given for you.

Know it! Note



3 Close

Summarize

Be sure students understand what the axes are representing. For example, if a person runs uphill at a *constant* speed, then a speed vs. time graph will be horizontal, but a distance or height vs. time graph will slant upward. Review what a graph of speed vs. time would look like for the following.

Someone is running downhill with an increasing speed. slant upward

Someone is running on a flat path at a decreasing speed. slant downward

ONGOING ASSESSMENT and INTERVENTION

***Diagnose Before* the Lesson**
Warm Up, TE

***Monitor During* the Lesson**
Check It Out! Exercises, SE
Questioning Strategies, TE

***Assess After* the Lesson**
Lesson Quiz, TE
Alternative Assessment, TE

Answers to *Think and Discuss*

1. Continuous; if you plot distinct points, you could always plot more points between any pair of points.
2. Possible answer: Speed related to time—a runner runs a race and keeps a steady pace for part of it.
3. See Additional Answers.

Learn It Online
Homework Help Online
Parent Resources Online

GUIDED PRACTICE

Vocabulary Apply the vocabulary from this lesson to answer each question.

continuous **1.** A ___?___ graph is made of connected lines or curves. (*continuous* or *discrete*)

discrete **2.** A ___?___ graph is made of only distinct points. (*continuous* or *discrete*)

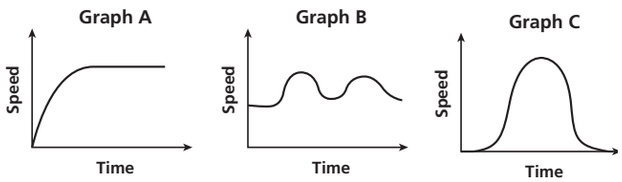
SEE EXAMPLE 1

Choose the graph that best represents each situation.

3. A person alternates between running and walking. B

4. A person gradually speeds up to a constant running pace. A

5. A person walks, gradually speeds up to a run, and then slows back down to a walk. C



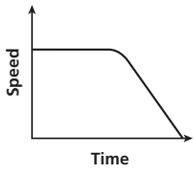
SEE EXAMPLE 2

6. Maxine is buying extra pages for her photo album. Each page holds exactly 8 photos. Sketch a graph to show the maximum number of photos she can add to her album if she buys 1, 2, 3, or 4 extra pages. Tell whether the graph is continuous or discrete.

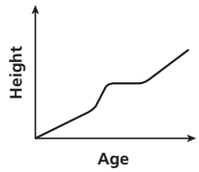
SEE EXAMPLE 3

Write a possible situation for each graph.

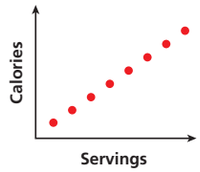
7.



8.



9.



PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
10–12	1
13	2
14–16	3

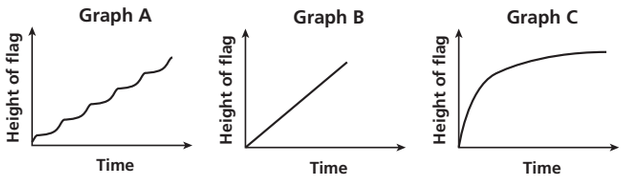
Extra Practice
See Extra Practice for more Skills Practice and Applications Practice exercises.

Choose the graph that best represents each situation.

10. A flag is raised up a flagpole quickly at the beginning and then more slowly near the top. C

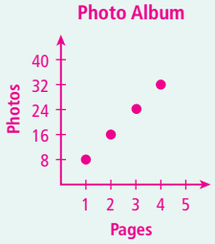
11. A flag is raised up a flagpole in a jerky motion, using a hand-over-hand method. A

12. A flag is raised up a flagpole at a constant rate of speed. B



Answers

6. discrete



7–9. Possible answers given.

7. A car is moving at a constant speed for several minutes. When it approaches a red light, it slows down and comes to a stop.

8. A plant grows steadily and then has a growth spurt during the rainy season. It remains the same height for a while and then grows steadily again.

9. The more servings you eat, the more Calories you consume.

3-1 Exercises

Assignment Guide

Assign *Guided Practice* exercises as necessary.

If you finished Examples **1–3**

Basic 10–18, 22–25
Average 10–25, 28
Advanced 10–28

Homework Quick Check
Quickly check key concepts.
Exercises: 10, 12, 13, 14, 18

Teaching Tip **Kinesthetic** For **Exercises 3–5,** you may want to have students act out the events happening in the situation to more easily match them to each graph.

Teaching Tip **Communicating Math** Point out to students that some graphs start at (0, 0), like the one in **Exercise 8,** which begins with a height of 0, while some start higher on the vertical axis, like the one in **Exercise 7,** which begins with a positive speed.

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them. Exercises 18–19, 22

Construct viable arguments and critique the reasoning of others. Exercises 20–21

Model with mathematics. Exercises 3–5, 7–12, 14–16, 26–28

Use appropriate tools strategically. Exercise 17

Learn It Online
State Resources Online

Data Collection To help students complete **Exercise 17,** see *Technology Lab Activities.*

Exercise 22 involves sketching a graph to describe a situation. This exercise prepares students for the Multi-Step Test Prep.

Answers

14–16, 18. Possible answers given.

14. A student is driven to school. The distance from home remains constant while the student is at school. Then the distance decreases during the ride back home.

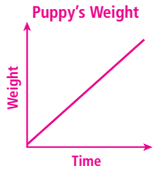
15. The cost of a long-distance phone call increases for the first few minutes and then increases at a slower rate for the remainder of the call.

16. Park attendance steadily declines over the course of a week due to bad weather.

18. The horse starts at the gate, increases speed very quickly, then keeps a fairly steady pace, and then increases speed again at the end of the race. After crossing the finish line the horse slows to a stop.

19–22. See Additional Answers.

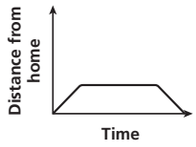
13. continuous



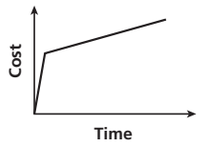
13. For six months, a puppy gained weight at a steady rate. Sketch a graph to illustrate the weight of the puppy during that time period. Tell whether the graph is continuous or discrete.

Write a possible situation for each graph.

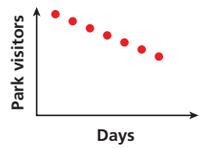
14.



15.



16.



17. Data Collection Use a graphing calculator and motion detector for the following. **17a–c. Check students' work.**

a. On a coordinate plane, draw a graph relating distance from a starting point walking at various speeds and time.

b. Using the motion detector as the starting point, walk away from the motion detector to make a graph on the graphing calculator that matches the one you drew.

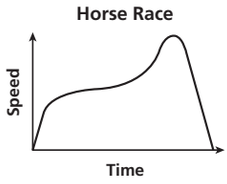
c. Compare your walking speeds to each change in steepness on the graph.

LINK

Sports

On November 1, 1938, the underdog Seabiscuit beat the heavily favored Triple-Crown winner War Admiral in a historic horse race at Pimlico Race Course in Baltimore, Maryland.

Sports The graph shows the speed of a horse during and after a race. Use it to describe the changing pace of the horse during the race.



19. Recreation You hike up a mountain path starting at 10 A.M. You camp overnight and then walk back down the same path at the same pace at 10 A.M. the next morning. On the same set of axes, graph the relationship between distance from the top of the mountain and the time of day for both the hike up and the hike down. What does the point of intersection of the graphs represent?

20. Critical Thinking Suppose that you sketched a graph of speed related to time for a brick that fell from the top of a building. Then you sketched a graph for speed related to time for a ball that was rolled down a hill and then came to rest. How would the graphs be the same? How would they be different?

21. Write About It Describe a real-life situation that could be represented by a distinct graph. Then describe a real-life situation that could be represented by a continuous graph.

MULTI-STEP TEST PREP

22. A rectangular pool that is 4 feet deep at all places is being filled at a constant rate.

a. Sketch a graph to show the depth of the water as it increases over time.

b. The side view of another swimming pool is shown. If the pool is being filled at a constant rate, sketch a graph to show the depth of the water as it increases over time.

(cl),© Bettmann/CORBIS; (bl), Sam Dudgeon/HMH

PRACTICE A

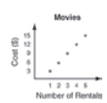
PRACTICE C

PRACTICE B

Choose the graph that best represents each situation.

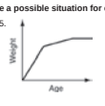
Graph A / Graph B / Graph C

1. A tomato plant grows taller at a steady pace. — Graph C
2. A tomato plant grows quickly at first, remains a constant height during a dry spell, then grows at a steady pace. — Graph B
3. A tomato plant grows at a slow pace, then grows rapidly with more sun and water. — Graph A
4. Lora has $15 to spend on movie rentals for the week. Each rental costs $3. Sketch a graph to show how much money she might spend on movies in a week. Tell whether the graph is continuous or discrete. — discrete

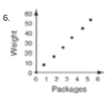


Write a possible situation for each graph.

5. Possible answer: A kitten gains weight quickly after birth, then more slowly, until it reaches its maximum weight.

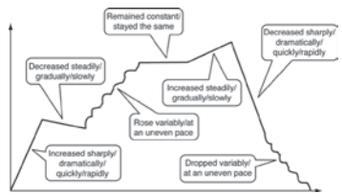


6. Possible answer: Each package weighs 10 pounds. The box can hold up to 60 pounds.



READING STRATEGIES

A relationship between variables is often shown as a continuous graph. Study the segments of the continuous graph below and the common phrases that describe them.



Answer the following.

1. Give two phrases that could describe this graph: Possible answers: rose slowly; increased gradually

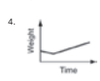
2. Sketch a graph that shows the speed of a car decreased dramatically and then remained constant.

Write the letter of the description that best matches each graph.

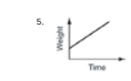
A. A baby lost a little weight, then gained weight slowly.
B. A baby gained weight at a rapid pace.
C. A baby gained weight at an uneven pace, then maintained a constant weight.

3. C

4. A



5. B



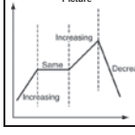
RETEACH

Graphs are a way to turn words into pictures. Be sure to read the graphs from left to right.

increasing	decreasing	stays the same
Other descriptions: rose, gained, grew	Other descriptions: fell, lessened, diminished	Other descriptions: constant, steady, continuous

You can divide the graph into sections every time the graph changes directions. Then label each section.

Picture

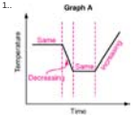


Words

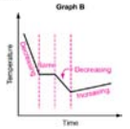
This graph increases, then stays constant, increases again, and finally decreases sharply.

Divide each graph into sections where the graph changes directions. Then label the sections as *increasing, decreasing,* or *same.*

1. Graph A



2. Graph B



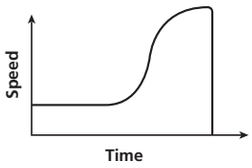
3. Which graph above shows that the air temperature fell steadily, leveled off, fell again, and then increased slightly? Graph B

TEST PREP

23. Which situation would NOT be represented by a discrete graph?

- (A) Amount of money earned based on the number of cereal bars sold
- (B) Number of visitors to a grocery store per day for one week
- (C) The amount of iced tea in a pitcher at a restaurant during the lunch hour
- (D) The total cost of buying 1, 2, or 3 CDs at the music store

24. Which situation is best represented by the graph?



- (F) A snowboarder starts at the bottom of the hill and takes a ski lift to the top.
- (G) A cruise boat travels at a steady pace from the port to its destination.
- (H) An object falls from the top of a building and gains speed at a rapid pace before hitting the ground.
- (J) A marathon runner starts at a steady pace and then runs faster at the end of the race before stopping at the finish line.

25. Short Response Marla participates in a triathlon consisting of swimming, biking, and running. Would a graph of Marla's speed during the triathlon be a continuous graph or a distinct graph? Explain.

Possible answer: The graph would be continuous because Marla is constantly moving at all times.

CHALLENGE AND EXTEND

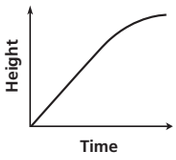
Pictured are three vases and graphs representing the height of water as it is poured into each of the vases at a constant rate. Match each vase with the correct graph.

A

B

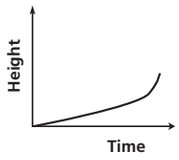
C

26.



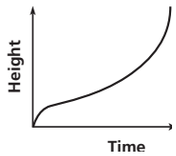
A

27.



C

28.



B

(all) Sam Dudgeon/HMH

PROBLEM SOLVING

Sketch a graph for the given situation. Tell whether the graph is discrete or continuous.

1. A giraffe is born 6 feet tall and continues to grow at a steady rate until it is fully grown.

continuous

2. The price of a used car is discounted $200 each week.

discrete

3. A city planner buys more buses as the population of her city grows.

discrete

4. Joseph is sky-diving. At first, he is free-falling rapidly and then he releases his parachute to slow his descent until he reaches the ground.

continuous

Choose the graph that best represents the situation.

5. Rebekah turns on the oven and sets it to 300 °F. She bakes a tray of cookies and then turns the oven off.

A Graph 1 C Graph 3
B Graph 2 D Graph 4

6. Leon puts ice cubes in his soup to cool it down before eating it.

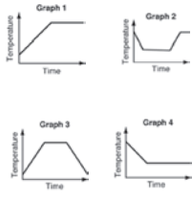
F Graph 1 H Graph 3
G Graph 2 J Graph 4

7. Barlee has the flu and her temperature rises slowly until it reaches 101 °F.

A Graph 1 C Graph 3
B Graph 2 D Graph 4

8. On a hot day, Karin walks into and out of an air-conditioned building.

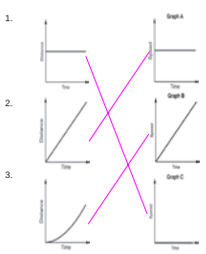
F Graph 1 H Graph 3
G Graph 2 J Graph 4



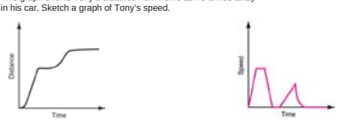
CHALLENGE

You have already seen graphs that show an object's distance related to time or an object's speed related to time. However, for any one object, the distance it travels and how long it takes to travel that distance are directly related to its speed. See if you can make the graphical connection between distance and speed.

For 1–3, draw a line from each graph of distance and time to its corresponding graph of distance and speed.



4. This graph shows Tony's distance from home as he drives away in his car. Sketch a graph of Tony's speed.



TEST PREP DOCTOR

For **Exercise 23,** students should look for a situation where both units can be broken into fractions. Time and the liquid measurement in choice **C** can both be fractional.

Journal

Write a situation in which the graph would contain either one vertical or one horizontal segment. Draw the graph.

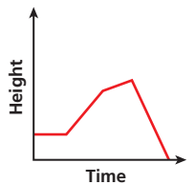
ALTERNATIVE ASSESSMENT

Have students write and graph a situation described by a distinct graph and a situation described by a continuous graph.

Power Presentations with PowerPoint®

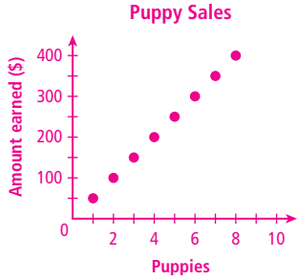
3-1 Lesson Quiz

1. Write a possible situation for the given graph.



Possible answer: The level of water in a bucket stays constant. A steady rain raises the level. The rain slows down. Someone dumps the bucket.

2. A pet store is selling puppies for $50 each. It has 8 puppies to sell. Sketch a graph for this situation.



Also available online

3-2 Organizer

Pacing: Traditional 1 day
Block $\frac{1}{2}$ day

Objectives: Identify functions.
Find the domain and range of relations and functions.

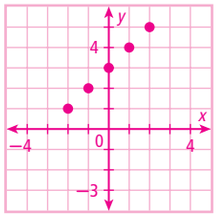
Online Edition
Tutorial Videos

Countdown

Power Presentations with PowerPoint®

Warm Up

1. Generate ordered pairs for the function $y = x + 3$ for $x = -2, -1, 0, 1$, and 2. Graph the ordered pairs. $(-2, 1)$, $(-1, 2)$, $(0, 3)$, $(1, 4)$, $(2, 5)$



Also available online

Math Humor

Q: Why did the *y*-variable leave the city?

A: He was more at home on the range.

COMMON CORE

CC.9-12.F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then $f(x)$ denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation $y = f(x)$.
CC.9-12.F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.*

Learn It Online
State Resources Online

COMMON CORE

3-2 Relations and Functions

CC.9-12.F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range.… *Also* **CC.9-12.F.IF.5***

Objectives
Identify functions.
Find the domain and range of relations and functions.

Vocabulary
relation
domain
range
function

Why learn this?
You can use a relation to show finishing positions and scores in a track meet.

Previously, you saw relationships represented by graphs. Relationships can also be represented by a set of ordered pairs, called a **relation**.

In the scoring system of some track meets, **first place** is worth 5 points, **second place** is worth 3 points, **third place** is worth 2 points, and **fourth place** is worth 1 point. This scoring system is a relation, so it can be shown as ordered pairs, $\{(1, 5), (2, 3), (3, 2), (4, 1)\}$. You can also show relations in other ways, such as tables, graphs, or *mapping diagrams*.

EXAMPLE 1 Showing Multiple Representations of Relations

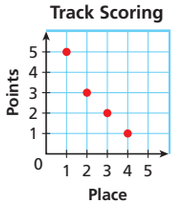
Express the relation for the track meet scoring system, $\{(1, 5), (2, 3), (3, 2), (4, 1)\}$, as a table, as a graph, and as a mapping diagram.

Table

Track Scoring	
Place	Points
1	5
2	3
3	2
4	1

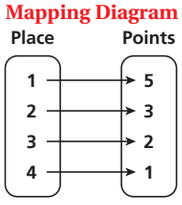
Write all x-values under "Place" and all y-values under "Points."

Graph



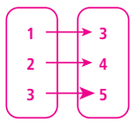
Use the x- and y-values to plot the ordered pairs.

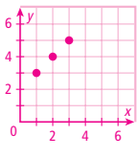
Mapping Diagram



Write all x-values under "Place" and all y-values under "Points." Draw an arrow from each x-value to its corresponding y-value.

1. | x | y |
|---|---|
| 1 | 3 |
| 2 | 4 |
| 3 | 5 |





CHECK IT OUT! **1.** Express the relation $\{(1, 3)\ (2, 4), (3, 5)\}$ as a table, as a graph, and as a mapping diagram.

The **domain** of a relation is the set of first coordinates (or *x*-values) of the ordered pairs. The **range** of a relation is the set of second coordinates (or *y*-values) of the ordered pairs. The domain of the track meet scoring system is {1, 2, 3, 4}. The range is {5, 3, 2, 1}.

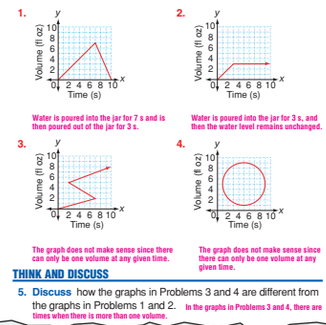
Alfo Foto Agency

1 Introduce

EXPLORATION

Relations and Functions

Each graph shows the volume of water in a jar over time. Describe a possible situation for each graph or explain why the graph does not make sense.



THINK AND DISCUSS

5. **Discuss** how the graphs in Problems 3 and 4 are different from the graphs in Problems 1 and 2. In the graphs in Problems 3 and 4, there are times when there is more than one volume.

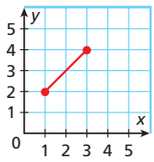
Motivate

Display the following chart and have students determine if the examples could contain negative numbers or fractions and decimals.

	Negative Numbers?	Fractions and Decimals?
Ages of Students	No	Yes
Temperatures in Antarctica	Yes	Yes
Number of Desks in Classrooms	No	No

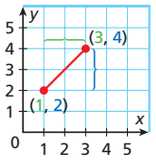
EXAMPLE 2 Finding the Domain and Range of a Relation

Give the domain and range of the relation.



The domain is all x-values from 1 through 3, inclusive.

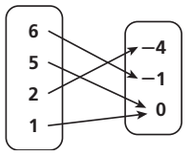
The range is all y-values from 2 through 4, inclusive.



D: $1 \le x \le 3$ R: $2 \le y \le 4$

CHECK IT OUT! **Give the domain and range of each relation.**

2a.



D: {6, 5, 2, 1};
R: {−4, −1, 0}

2b.

x	y
1	1
4	4
8	1

D: {1, 4, 8};
R: {1, 4}

A **function** is a special type of relation that pairs each domain value with exactly one range value.

EXAMPLE 3 Identifying Functions

Give the domain and range of each relation. Tell whether the relation is a function. Explain.

A

Field Trip	
Students x	Buses y
75	2
68	2
125	3

D: {75, 68, 125}
R: {2, 3}

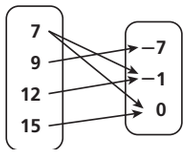
Even though 2 appears twice in the table, it is written only once when writing the range.

This relation is a function. Each domain value is paired with exactly one range value.

Writing Math

When there is a finite number of values in a domain or range, list the values inside braces.

B



Use the arrows to determine which domain values correspond to each range value.

D: {7, 9, 12, 15}
R: {−7, −1, 0}

This relation is not a function. Each domain value does not have exactly one range value. The domain value 7 is paired with the range values −1 and 0.

COMMON ERROR ALERT

Students may have difficulty remembering how to determine whether a relation is a function. Tell students to think of x as a person and y as a place. A person cannot be in more than one place at a time, but more than one person can be at the same place.

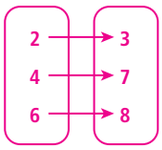
Power Presentations with PowerPoint®

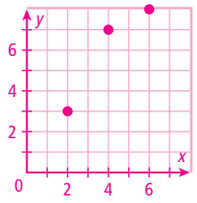
Additional Examples

Example 1

Express the relation {(2, 3), (4, 7), (6, 8)} as a table, as a graph, and as a mapping diagram.

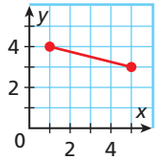
x	y
2	3
4	7
6	8





Example 2

Give the domain and range of the relation.



D: $1 \le x \le 5$

R: $3 \le y \le 4$

Also available online

2 Teach

Guided Instruction

Remind students that an ordered pair consists of (x, y). In **Examples 2** and **3,** encourage students to list the ordered pairs when given a mapping diagram, following one arrow at a time.

Teaching Tip **Communicating Math** Another way to write a domain or range is by using set-builder notation. For example, the domain $\{x | 1 \le x \le 3\}$ is read, "all x such that x is greater than or equal to 1 and x is less than or equal to 3."

Reaching All Learners

Through Cognitive Strategies

Show students the following to help them remember which values are in the domain and which are in the range.

- ***x*** and ***y*** are in alphabetical order.
- **D**omain and **r**ange are in alphabetical order.

So the x-values make up the domain, and the y-values make up the range.

MATHEMATICAL PRACTICES **Questioning Strategies**

EXAMPLE 1

- Which representation do you prefer? Why?

EXAMPLE 2

- When are the domain and range of a relation a continuous interval of values? When are they distinct numbers?

Power Presentations with PowerPoint®

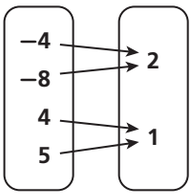
Additional Examples

Example 3

Give the domain and range of each relation. Tell whether the relation is a function. Explain.

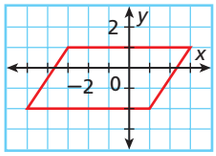
A. {(3, −2), (5, −1), (4, 0), (3, 1)} D: {3, 5, 4}; R: {−2, −1, 0, 1}; no; 3 is paired with −2 and 1.

B.



D: {−8, −4, 4, 5}; R: {2, 1}; yes; each domain element is paired with exactly one range element.

C.

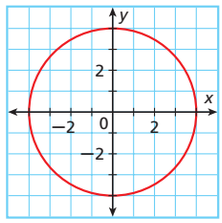


D: $-5 \le x \le 3$; R: $-2 \le y \le 1$; no; nearly all domain values have more than one range value.

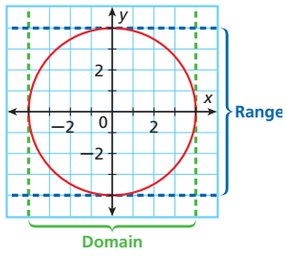
Also available online

Give the domain and range of each relation. Tell whether the relation is a function. Explain.

C



Draw lines to see the domain and range values.



D: $-4 \le x \le 4$ R: $-4 \le y \le 4$

x	4	0	0	−4
y	0	4	−4	0

To compare domain and range values, make a table using points from the graph.

This relation is not a function because there are several domain values that have more than one range value. For example, the domain value 0 is paired with both 4 and −4.

Helpful Hint

To find the domain and range of a graph, it may help to draw lines to see the *x*- and *y*-values.

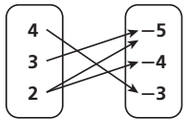
3a. D: {−6, −4, 1, 8}; R: {1, 2, 9}; yes; each domain value is paired with exactly one range value.

b. D: {2, 3, 4}; R: {−5, −4, −3}; no; the domain value 2 is paired with both −5 and −4.

CHECK IT OUT! **Give the domain and range of each relation. Tell whether the relation is a function. Explain.**

3a. $\{(8, 2), (-4, 1), (-6, 2), (1, 9)\}$

3b.



Student to Student — Functions

Eric Dawson
Boone High School

*I decide whether a list of ordered pairs is a function by looking at the **x-values**. If they're all different, then it's a function.*

(1, 6), (2, 5), (6, 5), (0, 8)
All different x-values
Function

(5, 6), (7, 2), (5, 8), (6, 3)
Same x-value (with different *y*-values)
Not a function

MATHEMATICAL PRACTICES

THINK AND DISCUSS

1. Describe how to tell whether a set of ordered pairs is a function.
2. Can the graph of a vertical line segment represent a function? Explain.
3. **GET ORGANIZED** Copy and complete the graphic organizer by explaining when a relation is a function and when it is not a function.

A relation is . . .	
A function if . . .	Not a function if . . .

Know it! Note

© Comstock Images/Getty

MATHEMATICAL PRACTICES Questioning Strategies

EXAMPLE 3

- Which of the displays (mapping diagram, table, or graph) is easiest for determining if a relation is a function? Why?

3 Close

Summarize

Ask students to name the four different ways relations are represented in this lesson. list of ordered pairs, table, graph, mapping diagram Write each of the following on the board. Have students find the domain and range and tell whether the relation is a function.

{(1, 4), (2, 4), (3, 5)}
D: {1, 2, 3}; R: {4, 5}; yes

{(1, 4), (2, 5), (2, 6)}
D: {1, 2}; R: {4, 5, 6}; no

ONGOING ASSESSMENT and INTERVENTION

***Diagnose Before* the Lesson**
Warm Up, TE

***Monitor During* the Lesson**
Check It Out! Exercises, SE
Questioning Strategies, TE

***Assess After* the Lesson**
Lesson Quiz, TE
Alternative Assessment, TE

Answers to *Think and Discuss*

1. A set of ordered pairs is a function if each *x*-value is assigned exactly one *y*-value.
2. No; the graph of a vertical line segment cannot represent a function because there is only one element in the domain and it is assigned many different elements in the range.
3. See Additional Answers.

Learn It Online
Homework Help Online
Parent Resources Online

GUIDED PRACTICE

Vocabulary Apply the vocabulary from this lesson to answer each question.

1. Use a mapping diagram to show a relation that is not a *function*.
2. The set of x-values for a relation is also called the __?__. (*domain* or *range*)

domain

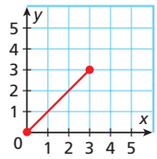
SEE EXAMPLE 1 Express each relation as a table, as a graph, and as a mapping diagram.

3. $\{(1, 1), (1, 2)\}$
4. $\{(-1, 1), (-2, \frac{1}{2}), (-3, \frac{1}{3}), (-4, \frac{1}{4})\}$
5. $\{(-1, 1), (-3, 3), (5, -5), (-7, 7)\}$
6. $\{(0, 0), (2, -4), (2, -2)\}$

SEE EXAMPLE 2 Give the domain and range of each relation.

7. $\{(-5, 7), (0, 0), (2, -8), (5, -20)\}$
8. $\{(1, 2), (2, 4), (3, 6), (4, 8), (5, 10)\}$
9.

x	3	5	2	8	6
y	9	25	4	81	36

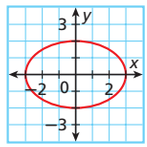
10. 

7. D: {−5, 0, 2, 5}; R: {−20, −8, 0, 7}
8. D: {1, 2, 3, 4, 5}; R: {2, 4, 6, 8, 10}
9. D: {2, 3, 5, 6, 8}; R: { 4, 9, 25, 36, 81}
10. D: $0 \le x \le 3$; R: $0 \le y \le 3$

SEE EXAMPLE 3 **Multi-Step** Give the domain and range of each relation. Tell whether the relation is a function. Explain.

11. $\{(1, 3), (1, 0), (1, -2), (1, 8)\}$
12. $\{(-2, 1), (-1, 2), (0, 3), (1, 4)\}$
13.

x	−2	−1	0	1	2
y	1	1	1	1	1

14. 

PRACTICE AND PROBLEM SOLVING

Independent Practice

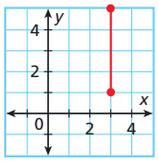
For Exercises	See Example
15–16	1
17–18	2
19–20	3

Extra Practice
See Extra Practice for more Skills Practice and Applications Practice exercises.

Express each relation as a table, as a graph, and as a mapping diagram.

15. $\{(-2, -4), (-1, -1), (0, 0), (1, -1), (2, -4)\}$
16. $\{(2, 1), (2, \frac{1}{2}), (2, 2), (2, 2\frac{1}{2})\}$

Give the domain and range of each relation.

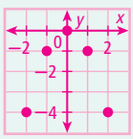
17. 
18.

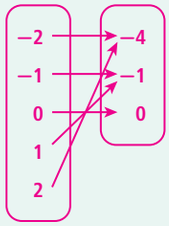
x	y
4	4
5	5
6	6
7	7
8	8

17. D: {3}; R: $1 \le y \le 5$
18. D: {4, 5, 6, 7, 8}; R: {4, 5, 6, 7, 8}

14. D: $-3 \le x \le 3$; R: $-2 \le y \le 2$; no; almost every domain value is paired with 2 range values.
15.

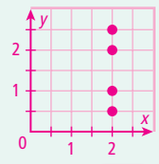
x	y
−2	−4
−1	−1
0	0
1	−1
2	−4

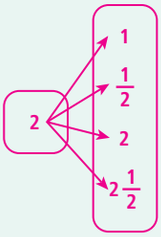




16.

x	y
2	1
2	$\frac{1}{2}$
2	2
2	$2\frac{1}{2}$





3-2 Exercises

Assignment Guide

Assign *Guided Practice* exercises as necessary.

If you finished Examples 1–3

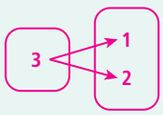
Basic 15–26, 29, 30, 32–35
Average 15–20, 22–24, 27–37
Advanced 15–20, 22, 24, 26–29, 31–38

Homework Quick Check
Quickly check key concepts.
Exercises: 16, 18, 20, 22, 24

Teaching Tip **Visual** Encourage students to write out some of the ordered pairs in **Exercise 17** to help them determine the domain and range.

Answers

1. Possible answer:



3–6. See Additional Answers.
11. D: {1}; R: {−2, 0, 3, 8}; no; the domain value 1 is paired with several different range values.
12. D: {−2, −1, 0, 1}; R: {1, 2, 3, 4}; yes; each domain value is paired with exactly one range value.
13. D: {−2, −1, 0, 1, 2}; R: {1}; yes; each element in the domain is paired with exactly one element in the range.

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them. Exercises 21–26, 29

Construct viable arguments and critique the reasoning of others. Exercises 27–28, 30–31

Look for and make use of structure. Exercises 3–20

Learn It Online
State Resources Online

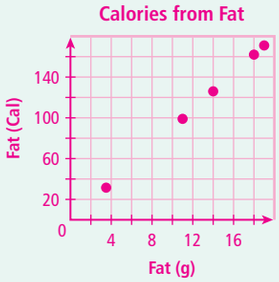
Teaching Tip **Geometry** In **Exercise 22,** remind students that the area of a square with side length s is s^2.

Answers

21. Yes; each domain value is paired with exactly one range value.

x	y
1	125
2	175
3	225
4	275

24a.



b. Yes; each domain value is paired with exactly one range value.

25.

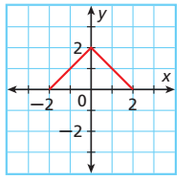
Hours x	Cost y
1	9
2	11
3	13
4	15
5	15

Yes; each domain value is paired with exactly one range value.

26. See Additional Answers.

Multi-Step Give the domain and range of each relation. Tell whether the relation is a function. Explain.

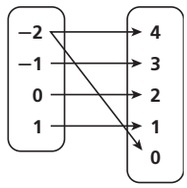
19.



D: $-2 \le x \le 2$; R: $0 \le y \le 2$

Yes; each domain value is paired with exactly one range value.

20.



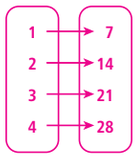
D: {−2, −1, 0, 1}; R: {0, 1, 2, 3, 4};

No; the domain value −2 is paired with 4 and 0.

21. **Consumer Application** An electrician charges a base fee of $75 plus $50 for each hour of work. Create a table that shows the amount the electrician charges for 1, 2, 3, and 4 hours of work. Let x represent the number of hours and y represent the amount charged for x hours. Is this relation a function? Explain.

23. Yes; each domain value is paired with exactly one range value.

22. **Geometry** Write a relation as a set of ordered pairs in which the x-value represents the side length of a square and the y-value represents the area of that square. Use a domain of 2, 4, 6, 9, and 11. {(2, 4), (4, 16), (6, 36), (9, 81), (11, 121)}



23. **Multi-Step** Create a mapping diagram to display the numbers of days in 1, 2, 3, and 4 weeks. Is this relation a function? Explain.

24. **Nutrition** The illustrations list the number of grams of fat and the number of Calories from fat for selected foods.

a. Create a graph for the relation between grams of fat and Calories from fat.

b. Is this relation a function? Explain.

Hamburger
Fat (g): 14
Fat (Cal): 126

Cheeseburger
Fat (g): 18
Fat (Cal): 162

Grilled chicken filet
Fat (g): 3.5
Fat (Cal): 31.5

Breaded chicken filet
Fat (g): 11
Fat (Cal): 99

Taco salad
Fat (g): 19
Fat (Cal): 171

27. No; if the number of range values is greater than the number of domain values, then there must be a domain value that is paired with more than one range value.

25. **Recreation** A shop rents canoes for a $7 equipment fee plus $2 per hour, with a maximum cost of $15 per day. Express the number of hours x and the cost y as a relation in table form, and find the cost to rent a canoe for 1, 2, 3, 4, and 5 hours. Is this relation a function? Explain.

26. **Health** You can burn about 6 Calories per minute bicycling. Let x represent the number of minutes bicycled, and let y represent the number of Calories burned.

a. Write ordered pairs to show the number of Calories burned by bicycling for 60, 120, 180, 240, or 300 minutes. Graph the ordered pairs.

b. Find the domain and range of the relation.

c. Does this graph represent a function? Explain.

27. **Critical Thinking** For a function, can the number of elements in the range be greater than the number of elements in the domain? Explain.

28a. False; a relation that has several of the same domain values paired with different range values is not a function.

28. **Critical Thinking** Tell whether each statement is true or false. If false, explain why.

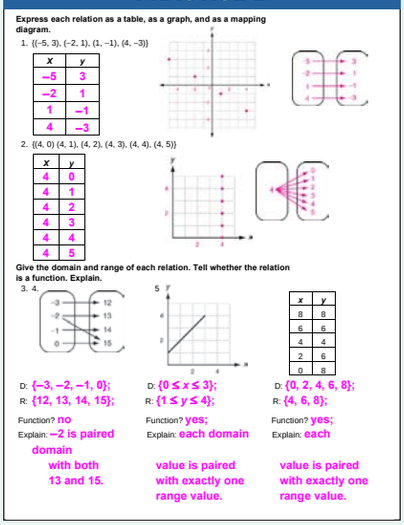
a. All relations are functions.

b. All functions are relations. true

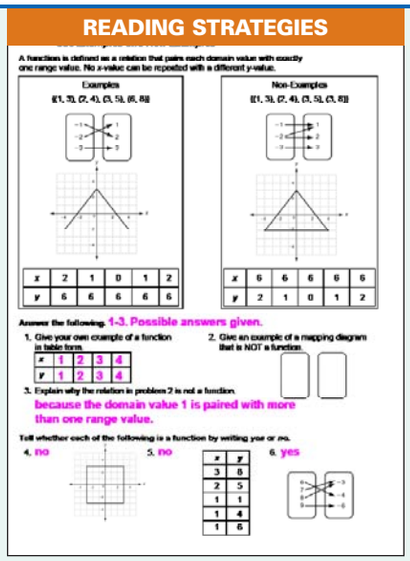
PRACTICE A

PRACTICE C

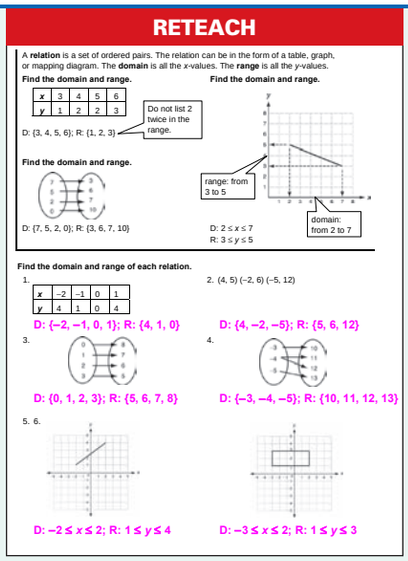
PRACTICE B



READING STRATEGIES



RETEACH



MULTI-STEP TEST PREP

29. **a.** The graph shows the amount of water being pumped into a pool over a 5-hour time period. Find the domain and range.

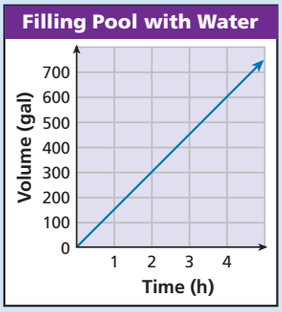
b. Does the graph represent a function? Explain.

c. Give the time and volume as ordered pairs at 2 hours and at 3 hours 30 minutes.

29a. D: $0 \le t \le 5$; R: $0 \le v \le 750$

b. Yes; each *x*-value is paired with exactly one *y*-value.

c. (2, 300); (3.5, 525)



30. ***///* ERROR ANALYSIS *///*** When asked whether the relation $\{(-4, 16), (-2, 4), (0, 0), (2, 4)\}$ is a function, a student stated that the relation is not a function because 4 appears twice. What error did the student make? How would you explain to the student why this relation is a function?

31. **Write About It** Describe a real-world situation for a relation that is NOT a function. Create a mapping diagram to show why the relation is not a function.

TEST PREP

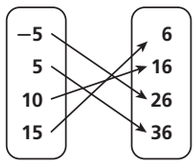
32. Which of the following relations is NOT a function?

Ⓐ $\{(6, 2), (-1, 2), (-3, 2), (-5, 2)\}$

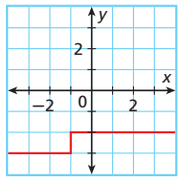
Ⓒ

x	3	5	7
y	1	15	30

Ⓑ

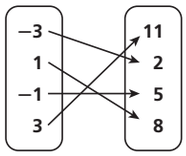


Ⓓ



33. Which is NOT a correct way to describe the function $\{(-3, 2), (1, 8), (-1, 5), (3, 11)\}$?

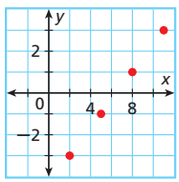
Ⓕ



Ⓗ Domain: $\{-3, 1, -1, 3\}$

Range: $\{2, 8, 5, 11\}$

Ⓖ



Ⓙ

x	*y*
−3	2
−1	5
1	8
3	11

Sam Dudgeon/HMH

MULTI-STEP TEST PREP **Exercise 29** involves finding the domain and range of a graph. This exercise prepares students for the Multi-Step Test Prep.

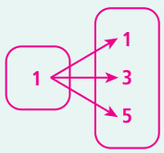
TEST PREP DOCTOR For **Exercise 32,** choices **A, B,** and **C** can be eliminated because no *x*-values are repeated.

Students who chose **F** in **Exercise 33** may not understand that in a mapping diagram, the domain and range values can be listed in an order other than that given. Students who chose **H** may not understand that in the domain and range, the *x*- and *y*-values can be listed out of order. Remind students that order does not matter.

Answers

30. The student thought that since there are two 4's, one element in the domain is paired with more than one element in the range. The relation is a function because each *x*-value is paired with exactly one *y*-value. A relation is a function even if a *y*-value has more than one *x*-value paired with it.

31. Possible answer: Packages being sent from a warehouse contain different numbers of items. One package contains 1 item, another package contains 3 items, and yet another package contains 5 items.



PROBLEM SOLVING

Identify the independent and dependent variables. Write a rule in function notation for each situation.

1. Each state receives electoral votes based on the number of representatives it has in the House of Representatives.

Representatives	2	4	6	8
Electoral Votes	4	6	8	10

I: number of representatives; D: number of electoral votes; $f(r) = r + 2$

2. Terry has 30 pieces of gum and gives 2 pieces to each of his friends.

I: number of friends; D: pieces of gum Terry has left; $f(x) = 30 - 2x$

3. Ronaldo is buying bacon that costs $4.29 per pound.

I: pounds of bacon; D: total price; $f(b) = 4.29b$

4. A personal trainer charges $50 for the first session and $40 for every session thereafter.

I: number of sessions; D: total cost; $f(s) = 50 + 40(s - 1)$

International travel and business require the conversion of American dollars into foreign currency. During part of 2005, one American dollar was worth 6 Croatian Kuna. Select the best answer.

5. An American bank wishes to convert *d* dollars into kuna. Which function rule describes the situation?

A $f(d) = \frac{d}{6}$ C $f(d) = \frac{6}{d}$

B $f(d) = 6d$ D $f(d) = d + 6$

6. A Croatian company already has $100,000 and is going to convert *k* kuna into dollars. Which function rule can be used to determine the total amount of American dollars this company will have?

F $f(x) = 100{,}000 + 6k$

G $f(x) = 100{,}000 + \frac{k}{6}$

H $f(x) = 100{,}000k + 6$

J $f(x) = 100{,}000 + \frac{6}{k}$

7. Macon has $100 and is thinking about converting some of it into kuna. What is a reasonable range for this situation?

A $0 \le y \le 6$ C $0 \le y \le 100$

B $0 \le y \le 16.7$ D $0 \le y \le 600$

8. Robin converts *x* dollars into *y* kuna. Which expression is the independent variable in this situation?

F *x* H 6*x*

G *y* J 6*y*

9. Jakov converts *n* kuna into *c* dollars. Which expression is the dependent variable in this situation?

A *n* C $\frac{n}{6}$

B *c* D $\frac{c}{6}$

CHALLENGE

A function is a correspondence, *f*, between two sets, *A* and *B*, such that each member of *A* is assigned exactly one member of *B*. The diagram shows a function as a dynamic process.

In Exercises 1–3, suppose that the input set, *A*, is the set of integers. Describe the range of each function.

1. *f*: Multiply each integer, *n*, by 5. multiples of 5: . . . , 15, 10, 5, 0, 5, 10, 15, . . .
2. *g*: Multiply each integer, *n*, by a fixed integer, *k*. multiples of *k*: *kn*
3. *h*: Divide the input integer, *n*, by 10 and write the remainder, *r*. 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

In Exercises 4–6, write a function rule that represents the function described.

4. *j*: the coordinate of the point halfway between each distinct pair of points on a number line — If *P* has coordinate *a*, *Q* has coordinate *b*, assign $\frac{a+b}{2}$.
5. *k*: the distance of each point from 0 — If *P* has coordinate *a*, assign $|a|$.
6. *m*: the length of the line segment determined by each distinct pair of points on the number line — If *P* has coordinate *a* and *Q* has coordinate *b*, assign $|a \quad b|$.

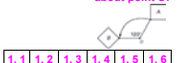
In Exercises 7 and 8, the domain of each function is the set of all squares.

7. a. Write a function, *P*, that gives the perimeter of any square. If the length of a side of square *X* is *s*, then *P* (square *X*) 4*s*.

b. Write a function, *A*, that gives the area of any square. If the length of a side of square *X* is *s*, then *A*(square *X*) s^2.

8. a. Using the diagram below, describe the function that moves the square from A to B. The function rotates square *A* 120 counterclockwise about point *O*.

b. What is the range of the function in part a? The set of all squares.



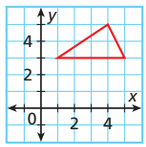
9. Suppose that you roll two number cubes. The sum of the numbers that are showing can be represented by a function.

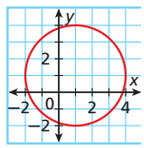
a. In the space at the right, make a table showing the members of the domain.

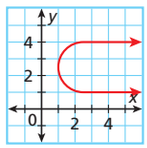
1, 1	1, 2	1, 3	1, 4	1, 5	1, 6
2, 1	2, 2	2, 3	2, 4	2, 5	2, 6
3, 1	3, 2	3, 3	3, 4	3, 5	3, 6
4, 1	4, 2	4, 3	4, 4	4, 5	4, 6
5, 1	5, 2	5, 3	5, 4	5, 5	5, 6
6, 1	6, 2	6, 3	6, 4	6, 5	6, 6

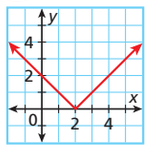
b. What is the range of the function? {2, 3, 4, 5, 6, 7, 8, 9, 10, 12}

34. Which graph represents a function?

Ⓐ 

Ⓑ 

Ⓒ 

Ⓓ 

35. Extended Response Use the table for the following.

x	−3	−1	0	1	3
y	5	7	9	11	13

a. Express the relation as ordered pairs.
b. Give the domain and range of the relation.
c. Does the relation represent a function? Explain your answer.

35a. {(−3, 5), (−1, 7), (0, 9), (1, 11), (3, 13)}

b. D: {−3, −1, 0, 1, 3}; R: {5, 7, 9, 11, 13}

Yes; each domain value is paired with exactly one range value.

CHALLENGE AND EXTEND

36. What values of a make the relation $\{(a, 1), (2, 3), (4, 5)\}$ a function? Explain.

37. What values of b make the relation $\{(5, 6), (7, 8), (9, b)\}$ a function? Explain.

38. The *inverse* of a relation is created by interchanging the x- and y- coordinates of each ordered pair in the relation.

a. Find the inverse of the following relation: $\{(-2, 5), (0, 4), (3, -8), (7, 5)\}$.
b. Is the original relation a function? Why or why not? Is the inverse of the relation a function? Why or why not?
c. The statement "If a relation is a function, then the inverse of the relation is also a function" is sometimes true. Give an example of a relation and its inverse that are both functions. Then give an example of a relation and its inverse that are both not functions.

Journal

Have students explain how to determine whether a graphed relation is a function.

ALTERNATIVE ASSESSMENT

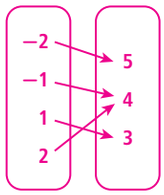
Have students write a list of four ordered pairs. Instruct them to write this relation as a table, a graph, and a mapping diagram. Have them find the domain and range and explain whether their relation is a function.

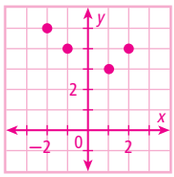
Power Presentations with PowerPoint®

3-2 Lesson Quiz

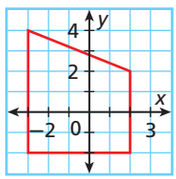
1. Express the relation $\{(-2, 5), (-1, 4), (1, 3), (2, 4)\}$ as a table, as a graph, and as a mapping diagram.

x	y
−2	5
−1	4
1	3
2	4



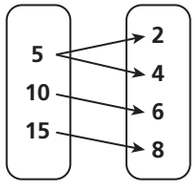


2. Give the domain and range of the relation.



D: $-3 \le x \le 2$; R: $-2 \le y \le 4$

3. Give the domain and range of the relation. Tell whether the relation is a function. Explain.



D: {5, 10, 15}; R: {2, 4, 6, 8}; no; 5 is paired with 2 and 4.

Also available online

Answers

36. All real numbers except 2 and 4; values in the domain cannot repeat.

37. All real numbers; values in the range can repeat.

38a. {(5, −2), (4, 0), (−8, 3), (5, 7)}

b. The original relation is a function because each domain value is paired with exactly one range value. The inverse is not a function because a domain value has more than one range value paired with it.

38c. Possible answer: Relation: {(1, 2), (2, 3), (3, 4)}; Inverse: {(2, 1), (3, 2), (4, 3)}; Relation: {(1, 1), (1, 2), (2, 2), (2, 4)}; Inverse: {(1, 1), (2, 1), (2, 2), (4, 2)}

3-2 Algebra LAB

The Vertical-Line Test

Use with Relations and Functions

The *vertical-line test* can be used to visually determine whether a graphed relation is a function.

MATHEMATICAL PRACTICES **Look for and express regularity in repeated reasoning.**

CC.9-12.F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range….

Activity

1. Look at the values in Table 1. Is every x-value paired with exactly one y-value? If not, what x-value(s) are paired with more than one y-value?
   yes
2. Is the relation a function? Explain.
   Yes; each x-value has exactly one y-value.
3. Graph the points from the Table 1. Draw a vertical line through each point of the graph. Does any vertical line touch more than one point?

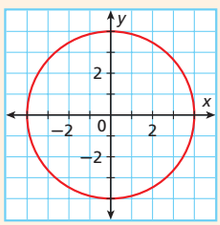
Table 1	
x	y
−2	−5
−1	−3
0	−1
1	1
2	3
3	5

4. Look at the values in Table 2. Is every x-value paired with exactly one y-value? If not, what x-value(s) are paired with more than one y-value?
   No; the x-value 1 has two y-values: 2 and 4.
5. Is the relation a function? Explain.
   No; an x-value has more than one y-value.
6. Graph the points from the Table 2. Draw a vertical line through each point of the graph. Does any vertical line touch more than one point?
7. What is the x-value of the two points that are on the same vertical line? Is that x-value paired with more than one y-value? **1; yes**
8. Write a statement describing how to use a vertical line to tell if a relation is a function. This is called the vertical-line test. **Possible answer: A relation is a function if there are no vertical lines that intersect the graph at more than one point.**
9. Why does the vertical-line test work?
   The vertical-line test works because it shows if there is more than 1 y-value for an x-value.

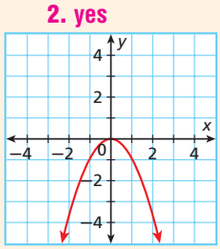
Table 2	
x	y
−2	−3
1	4
0	5
1	2
2	3
3	5

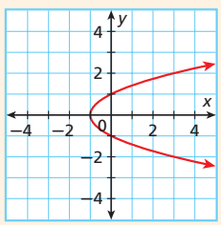
Try This

Use the vertical-line test to determine whether each relation is a function. If a relation is not a function, list two ordered pairs that show the same x-value with two different y-values.

1. 

1. no; possible answer: (0, −4), (0, 4)

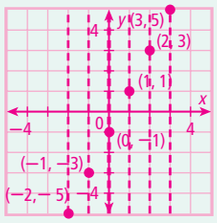
2. **2. yes**


3. 

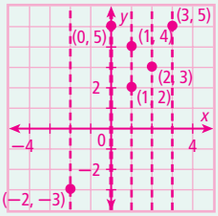
3. no; possible answer: (3, 2), (3, −2)

Answers to Activity

3. no



6. yes



Algebra LAB **Organizer**

Use with the lesson *Relations and Functions*

Pacing:
Traditional $\frac{1}{2}$ day
Block $\frac{1}{4}$ day

Objective: Determine if a relation is a function.

INTERACTIVE **Online Edition**
Graphing Calculator

Countdown

Lab Resources

Algebra LAB ***Algebra Lab Activities***
Lab Recording Sheet

Teach

Discuss

A vertical line has the same x-value for each y-value.

Alternative Approach

To perform the vertical-line test with a graphing calculator, press GRAPH 2nd DRAW PRGM, choose 4:Vertical, and use arrow keys to move the vertical line across the graph.

Close

Key Concept

If any vertical line touches the graph no more than once, then the graphed relation is a function.

Assessment

Journal Have students graph a relation that is not a function and explain why it is not a function.

MATHEMATICAL PRACTICES

Look for and express regularity in repeated reasoning.

COMMON CORE

CC.9-12.F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.

Learn It Online
State Resources Online

Organizer

Use with the lesson *Writing Functions*

Pacing:
Traditional $\frac{1}{2}$ day
Block $\frac{1}{4}$ day

Objective: Use models to represent algebraic relationships.

Online Edition

Countdown

Resources

Algebra Lab Activities
Lab Recording Sheet

Teach

Discuss

The expression for the number of blocks is *length of tabletop plus product of 2 and height of base*. Only the height of the base changes.

Alternative Approach

Use square tiles to model the pattern.

Close

Key Concept

You can use models to find a pattern, and algebra to describe it.

Assessment

Journal Sketch a model of a table with a length of 6 and a base of 4 columns. Extend the pattern by three terms by adding rows of blocks to the table's base. Have students write expressions for each term of the pattern.

MATHEMATICAL PRACTICES

Look for and express regularity in repeated reasoning.

Learn It Online
State Resources Online

3-3 Algebra LAB

Use with Writing Functions

Model Variable Relationships

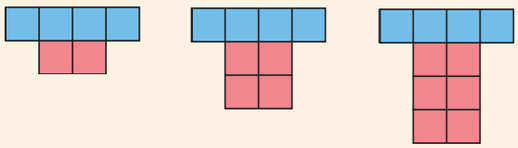
You can use models to represent an algebraic relationship. Using these models, you can write an algebraic expression to help describe and extend patterns.

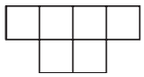
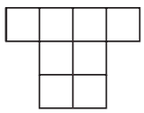
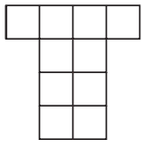
The diagrams below represent the side views of tables. Each has a tabletop and a base. Copy and complete the chart using the pattern shown in the diagrams.

MATHEMATICAL PRACTICES

Look for and express regularity in repeated reasoning.

Tabletop →
Base →



TERM NUMBER	FIGURE	DESCRIPTION OF FIGURE	EXPRESSION FOR NUMBER OF BLOCKS	VALUE OF TERM (NUMBER OF BLOCKS)	ORDERED PAIR
1		length of tabletop = 4 height of base = 1	$4 + (2)1$	6	$(1, 6)$
2		length of tabletop = 4 height of base = 2	$4 + (2)2$	8	$(2, 8)$
3		length of tabletop = 4 height of base = 3	$4 + (2)3$	10	$(3, 10)$
4		length of tabletop = 4 height of base = 4	$4 + (2)4$	12	$(4, 12)$
5		length of tabletop = 4 height of base = 5	$4 + (2)5$	14	$(5, 14)$
n		length of tabletop = 4 height of base = n	$4 + 2n$		$(n, 4 + 2n)$

2. The height of the base is 1 block, and a total of 6 blocks are in the table.

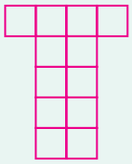
3. Yes; if the height of the table is 10 blocks, a total of 24 blocks will be in the table: $4 + 2(10) = 24$.

Try This

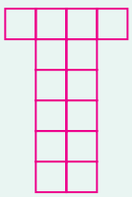
1. Explain why you must multiply the height of the base by 2. There are 2 base columns.
2. What does the ordered pair (1, 6) mean?
3. Does the ordered pair (10, 24) belong in this pattern? Why or why not?
4. Which expression from the table describes how you would find the total number of blocks for any term number n? $4 + 2n$
5. Use your rule to find the 25th term in this pattern. 54

Answers to Activity

4.



5.



COMMON CORE

3-3 Writing Functions

CC.9-12.F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. *Also* **CC.9-12.F.IF.1, CC.9-12.F.IF.5*, CC.9-12.A.CED.3*, CC.9-12.F.BF.1*, CC.9-12.F.LE.2***

Objectives
Identify independent and dependent variables.

Write an equation in function notation and evaluate a function for given input values.

Vocabulary
independent variable
dependent variable
function rule
function notation

Why learn this?
You can use a function rule to calculate how much money you will earn for working specific amounts of time.

Suppose Tasha baby-sits and charges $5 per hour.

Time Worked (h) x	1	2	3	4
Amount Earned ($) y	5	10	15	20

The amount of money Tasha earns is $5 times the number of hours she works. Write an equation using two different variables to show this relationship.

Amount earned is $5 times the number of hours worked.

$y = 5 \cdot x$

Tasha can use this equation to find how much money she will earn for any number of hours she works.

EXAMPLE 1 Using a Table to Write an Equation

Determine a relationship between the x- and y-values. Write an equation.

x	1	2	3	4
y	−2	−1	0	1

Step 1 List possible relationships between the first x- and y-values.

$1 - 3 = -2$ or $1(-2) = -2$

Step 2 Determine if one relationship works for the remaining values.

$2 - 3 = -1$ ✓ $\quad 2(-2) \neq -1$ ✗

$3 - 3 = 0$ ✓ $\quad 3(-2) \neq 0$ ✗

$4 - 3 = 1$ ✓ $\quad 4(-2) \neq 1$ ✗

The first relationship works. The value of y is 3 less than x.

Step 3 Write an equation.

$y = x - 3$ *The value of y is 3 less than x.*

CHECK IT OUT!

1. Determine a relationship between the x- and y-values in the relation $\{(1, 3), (2, 6), (3, 9), (4, 12)\}$. Write an equation. $y = 3x$

The equation in Example 1 describes a function because for each x-value (input), there is only one y-value (output).

RubberBall/Alamy

3-3 Organizer

Pacing: Traditional 2 days
Block 1 day

Objectives: Identify independent and dependent variables.

Write an equation in function notation and evaluate a function for given input values.

Algebra Lab
In *Algebra Lab Activities*

Online Edition
Tutorial Videos

Countdown

Power Presentations with PowerPoint®

Warm Up

Evaluate each expression for $a = 2$, $b = -3$, and $c = 8$.

1. $a + 3c$ 26
2. $ab - c$ −14
3. $\frac{1}{2}c + b$ 1
4. $4c - b$ 35
5. $b^a + c$ 17

Also available online

Math Humor

Q: Why did the x-variable move back home?

A: She was more comfortable in her own domain.

1 Introduce

EXPLORATION

Writing Functions

Dan has $100 in his savings account. His grandmother promises to deposit $25 into the account for each day Dan helps her during his summer vacation.

1. Complete the table to show how much money will be in Dan's account for each given number of days Dan works for his grandmother.
2. How much money will be in Dan's account if he works x days? $100 + 25x$
3. Write an equation that shows the relationship between x (the number of days Dan works) and y (the total in his account). $y = 100 + 25x$

Days x	Process	Total in Account ($) y
0	$100 + 25(0)$	100
1	$100 + 25(1)$	125
2	$100 + 25(2)$	150
3	$100 + 25(3)$	175
5	$100 + 25(5)$	225
10	$100 + 25(10)$	350

THINK AND DISCUSS

4. **Discuss** whether the above relationship is a function. It is a function since there is only one y-value for each x-value.
5. **Explain** how you can use your equation to find the total in Dan's account if he works 20 days. Substitute 20 for x in the equation.

Motivate

ENGLISH LANGUAGE LEARNERS

Have students describe the meaning of the following phrases: Possible answers:

- independently wealthy doesn't need a job
- Independence Day day of freedom
- working independently doesn't need help
- dependent child needs a parent
- insulin dependent needs insulin
- dependent on friends needs friends

Explorations and answers are provided in your online resources.

COMMON CORE

CC.9-12.F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
CC.9-12.F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range….
Also **CC.9-12.F.IF.5*, CC.9-12.A.CED.3*, CC.9-12.F.BF.1*, CC.9-12.F.LE.2***

Learn It Online
State Resources Online

Power Presentations with PowerPoint®

Additional Examples

Example 1

Determine a relationship between the *x*- and *y*-values. Write an equation. $y = \frac{x}{5}$

x	5	10	15	20
y	1	2	3	4

Example 2

Identify the independent and dependent variables in each situation.

A. A painter must measure a room before deciding how much paint to buy. ind.: measurement of room; dep.: amount of paint

B. The height of a candle decreases for every hour it burns. ind.: time; dep.: height of candle

C. A veterinarian must weigh an animal before determining the amount of medication. ind.: weight of animal; dep.: amount of medication

Also available online

Questioning Strategies

EXAMPLE 1

- As the *x*-value is increasing, what is happening to the *y*-value?
- What situation could this table of numbers represent?

EXAMPLE 2

- What is another situation where you would have independent and dependent variables?

The **input** of a function is the **independent variable**. The **output** of a function is the **dependent variable**. The value of the dependent variable *depends* on, or is a function of, the value of the independent variable. For Tasha, the amount she earns depends on, or is a function of, the amount of time she works.

EXAMPLE 2 Identifying Independent and Dependent Variables

Identify the independent and dependent variables in each situation.

A **In the winter, more electricity is used when the temperature goes down, and less is used when the temperature rises.**

The **amount of electricity** used *depends on* the **temperature**.

Dependent: **amount of electricity** Independent: **temperature**

B **The cost of shipping a package is based on its weight.**

The **cost** of shipping a package *depends on* its **weight**.

Dependent: **cost** Independent: **weight**

C **The faster Ron walks, the quicker he gets home.**

The **time** it takes Ron to get home *depends on* the **speed** he walks.

Dependent: **time** Independent: **speed**

Helpful Hint

There are several different ways to describe the variables of a function.

Independent Variable	Dependent Variable
x-values	*y*-values
Domain	Range
Input	Output
x	*f*(*x*)

CHECK IT OUT! **Identify the independent and dependent variables in each situation.**

2a. A company charges $10 per hour to rent a jackhammer.

2b. Apples cost $0.99 per pound.

2a. ind.: time; dep.: cost

ind.: pounds; dep.: cost

An algebraic expression that defines a function is a **function rule**. $5 \cdot x$ in the equation about Tasha's earnings is a function rule.

If *x* is the independent variable and *y* is the dependent variable, then **function notation** for *y* is *f*(*x*), read "*f* of *x*," where *f* names the function. When an equation in two variables describes a function, you can use function notation to write it.

The dependent variable is a function of the independent variable.

y is a function of *x*.

$y = f(x)$

Since $y = f(x)$, Tasha's earnings, $y = 5x$, can be rewritten in function notation by substituting *f*(*x*) for *y*: $f(x) = 5x$. Sometimes functions are written using *y*, and sometimes functions are written using *f*(*x*).

EXAMPLE 3 Writing Functions

Identify the independent and dependent variables. Write an equation in function notation for each situation.

A **A lawyer's fee is $200 per hour for her services.**

The **fee** for the lawyer depends on how many **hours** she works.

Dependent: **fee** Independent: **hours**

Let h represent the number of hours the lawyer works.

The function for the lawyer's fee is $f(h) = 200h$.

2 Teach

Guided Instruction

If students have a hard time finding a relationship in **Example 1,** have them first test the four basic operations. (Is the same value added to each *x*, subtracted from each *x*, etc.?) If those don't work, then it may be a square, square root, or combination of operations. In **Examples 2** and **3,** encourage students to rewrite each statement using the words *depends on* before choosing which variables are independent and dependent.

Reaching All Learners

Through Communication

Designate a group of four students as the "function machine." This group must create a secret function rule. Other students give input values to the group, and the group must state the output value. The other students must guess the secret function rule. Students can explain how they deduced the secret function rule.

Through Graphic Organizers

Divide students into groups and have them write examples of dependent relationships in various formats.

For example:

- I get cranky when I don't eat.
- If I don't work, then I don't get paid.

Independent	Dependent
• when I don't eat • if I don't work	• I get cranky • then I don't get paid

Identify the independent and dependent variables. Write an equation in function notation for each situation.

B **The admission fee to a local carnival is $8. Each ride costs $1.50.**

The **total cost** depends on **the number of rides** ridden, plus $8.

Dependent: **total cost** Independent: **number of rides**

Let r represent the number of rides ridden.

The function for the total cost of the carnival is $f(r) = 1.50r + 8$.

CHECK IT OUT! **Identify the independent and dependent variables. Write an equation in function notation for each situation.**

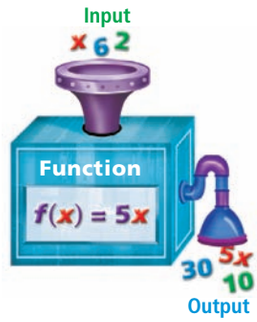
3a. Steven buys lettuce that costs $1.69/lb.

3a. ind.: pounds; dep.: cost; $f(x) = 1.69x$

3b. An amusement park charges a $6.00 parking fee plus $29.99 per person. ind.: people; dep.: cost; $f(x) = 6 + 29.99x$

You can think of a function as an **input-output** machine. For Tasha's earnings, $f(x) = 5x$, if you input a value x, the output is $5x$.

If Tasha wanted to know how much money she would earn by working 6 hours, she could input 6 for x and find the output. This is called *evaluating the function.*



EXAMPLE 4 Evaluating Functions

Evaluate each function for the given input values.

A **For $f(x) = 5x$, find $f(x)$ when $x = 6$ and when $x = 7.5$.**

$f(x) = 5x$

$f(6) = 5(6)$ *Substitute 6 for x.*

$= 30$ *Simplify.*

$f(x) = 5x$

$f(7.5) = 5(7.5)$ *Substitute 7.5 for x.*

$= 37.5$ *Simplify.*

Reading Math

Functions can be named with any letter; *f*, *g*, and *h* are the most common. You read $f(6)$ as "*f* of 6," and $g(2)$ as "*g* of 2."

B **For $g(t) = 2.30t + 10$, find $g(t)$ when $t = 2$ and when $t = -5$.**

$g(t) = 2.30t + 10$

$g(2) = 2.30(2) + 10$

$= 4.6 + 10$

$= 14.6$

$g(t) = 2.30t + 10$

$g(-5) = 2.30(-5) + 10$

$= -11.5 + 10$

$= -1.5$

C **For $h(x) = \frac{1}{2}x - 3$, find $h(x)$ when $x = 12$ and when $x = -8$.**

$h(x) = \frac{1}{2}x - 3$

$h(12) = \frac{1}{2}(12) - 3$

$= 6 - 3$

$= 3$

$h(x) = \frac{1}{2}x - 3$

$h(-8) = \frac{1}{2}(-8) - 3$

$= -4 - 3$

$= -7$

CHECK IT OUT! **Evaluate each function for the given input values.**

4a. For $h(c) = 2c - 1$, find $h(c)$ when $c = 1$ and $c = -3$.

4a. $h(1) = 1$; $h(-3) = -7$

4b. For $g(t) = \frac{1}{4}t + 1$, find $g(t)$ when $t = -24$ and $t = 400$.

$g(-24) = -5$; $g(400) = 101$

Teaching Tip **Auditory** Tell students that the ***in***put is the ***in***dependent variable. ENGLISH LANGUAGE LEARNERS

COMMON ERROR ALERT

Students might mistake the notation $f(x)$ as meaning *f* times *x*. Remind them that $f(x)$ is the notation of a function and, in a function, is interchangeable with *y*.

Power Presentations with PowerPoint®

Additional Examples

Example 3

Identify the independent and dependent variables. Write an equation in function notation for each situation.

A. A math tutor charges $35 per hour. ind.: time; dep.: cost; $f(h) = 35h$

B. A fitness center charges a $100 initiation fee plus $40 per month. ind.: number of months; dep.: total cost; $f(m) = 100 + 40m$

Example 4

Evaluate each function for the given input values.

A. For $f(x) = 3x + 2$, find $f(x)$ when $x = 7$ and when $x = -4$. $f(7) = 23$; $f(-4) = -10$

B. For $g(t) = 1.5t - 5$, find $g(t)$ when $t = 6$ and when $t = -2$. $g(6) = 4$; $g(-2) = -8$

C. For $h(r) = \frac{1}{3}r + 2$, find $h(r)$ when $r = 600$ and when $r = -12$. $h(600) = 202$; $h(-12) = -2$

Also available online

MATHEMATICAL PRACTICES Questioning Strategies

EXAMPLE 3

- How could you write the equation without using function notation?
- Does $f(x)$ represent domain or range?

EXAMPLE 4

- How is evaluating a function like solving an equation? How is it different?

Teaching Tip **Inclusion** Encourage students to identify the dependent and independent variables in **Example 5** before attempting to write the function.

Power Presentations with PowerPoint®

Additional Examples

Example 5

Joe has enough money to purchase 1, 2, or 3 DVDs at $15.00 each.

Write a function to describe the situation. Find the reasonable domain and range for the function. $f(x) = 15x$; D: {1, 2, 3}; R: {15, 30, 45}

Also available online

MATHEMATICAL PRACTICES **Questioning Strategies**

EXAMPLE 5

- What do you look for when finding the reasonable domain?
- How do you find the reasonable range once you have determined the reasonable domain?

When a function describes a real-world situation, every real number is not always reasonable for the domain and range. For example, a number representing the length of an object cannot be negative, and only whole numbers can represent a number of people.

EXAMPLE 5 Finding the Reasonable Domain and Range of a Function

Manuel has already sold $20 worth of tickets to the school play. He has 4 tickets left to sell at $2.50 per ticket. Write a function to describe how much money Manuel can collect from selling tickets. Find the reasonable domain and range for the function.

Money collected from ticket sales	is	$2.50	per	ticket	plus	the $20 already sold.
$f(x)$	=	$2.50	•	x	+	20

If he sells x more tickets, he will have collected $f(x) = 2.50x + 20$ dollars.

Manuel has only 4 tickets left to sell, so he could sell 0, 1, 2, 3, or 4 tickets. A reasonable domain is {0, 1, 2, 3, 4}.

Substitute these values into the function rule to find the range values.

x	0	1	2	3	4
$f(x)$	2.50(0) + 20 = 20	2.50(1) + 20 = 22.50	2.50(2) + 20 = 25	2.50(3) + 20 = 27.50	2.50(4) + 20 = 30

The reasonable range for this situation is {$20, $22.50, $25, $27.50, $30}.

CHECK IT OUT! **5.** The settings on a space heater are the whole numbers from 0 to 3. The total number of watts used for each setting is 500 times the setting number. Write a function to describe the number of watts used for each setting. Find the reasonable domain and range for the function.
$f(x) = 500x$; D: {0, 1, 2, 3}; R: {0, 500, 1000, 1500}

MATHEMATICAL PRACTICES

THINK AND DISCUSS

1. When you input water into an ice machine, the output is ice cubes. Name another real-world object that has an input and an output.
2. How do you identify the independent and dependent variables in a situation?
3. Explain how to find reasonable domain values for a function.
4. **GET ORGANIZED** Copy and complete the graphic organizer. Use the function $y = x + 3$ and the domain $\{-2, -1, 0, 1, 2\}$.

Know it! Note

3 Close

Summarize

Remind students that you need to know the value of the independent variable to be able to determine the value of the dependent variable. Then have students write an equation for the following situations using function notation.

A cable TV service charges $50 for hookup and $30 per month. $f(m) = 50 + 30m$

A library charges $0.25 for every day that a book is late. $f(d) = 0.25d$

ONGOING ASSESSMENT and INTERVENTION

***Diagnose Before* the Lesson**
Warm Up, TE

***Monitor During* the Lesson**
Check It Out! Exercises, SE
Questioning Strategies, TE

***Assess After* the Lesson**
Lesson Quiz, TE
Alternative Assessment, TE

Answers to *Think and Discuss*

Possible answers:

1. a toaster
2. If one data set depends on the other, the first data set represents the dependent variable, and the other data set represents the independent variable.
3. Find what makes sense for the real-world situation. Time, for example, would not be negative.
4. See Additional Answers.

Learn It Online
Homework Help Online
Parent Resources Online

GUIDED PRACTICE

Vocabulary Apply the vocabulary from this lesson to answer each question.

1. The output of a function is the ___?___ variable. (*independent* or *dependent*) dependent
2. An algebraic expression that defines a function is a ___?___. (*function rule* or *function notation*) function rule

SEE EXAMPLE 1

Determine a relationship between the x- and y-values. Write an equation.

3.

x	1	2	3	4
y	−1	0	1	2

$y = x - 2$

4. $\{(1, 4), (2, 7), (3, 10), (4, 13)\}$ $y = 3x + 1$

SEE EXAMPLE 2

Identify the independent and dependent variables in each situation.

5. A small-size bottle of water costs \$1.99 and a large-size bottle of water costs \$3.49.
ind.: size of bottle; dep.: cost of water
6. An employee receives 2 vacation days for every month worked.
ind.: months worked; dep.: vacation days available

SEE EXAMPLE 3

Identify the independent and dependent variables. Write an equation in function notation for each situation. ind.: hours; dep.: cost; $f(h) = 75h$

7. An air-conditioning technician charges customers \$75 per hour.
8. An ice rink charges \$3.50 for skates and \$1.25 per hour.
ind.: hours; dep.: cost; $f(x) = 3.50 + 1.25x$

SEE EXAMPLE 4

Evaluate each function for the given input values.

9. For $f(x) = 7x + 2$, find $f(x)$ when $x = 0$ and when $x = 1$. $f(0) = 2$; $f(1) = 9$
10. For $g(x) = 4x - 9$, find $g(x)$ when $x = 3$ and when $x = 5$. $g(3) = 3$; $g(5) = 11$
11. For $h(t) = \frac{1}{3}t - 10$, find $h(t)$ when $t = 27$ and when $t = -15$. $h(27) = -1$; $h(-15) = -15$

SEE EXAMPLE 5

12. A construction company uses beams that are 2, 3, or 4 meters long. The measure of each beam must be converted to centimeters. Write a function to describe the situation. Find the reasonable domain and range for the function. (*Hint*: 1 m = 100 cm) $f(\ell) = 100\ell$; D: {2, 3, 4}; R: {200, 300, 400}

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
13–14	1
15–16	2
17–19	3
20–22	4
23	5

Extra Practice

See Extra Practice for more Skills Practice and Applications Practice exercises.

Determine a relationship between the x- and y-values. Write an equation.

13.

x	1	2	3	4
y	−2	−4	−6	−8

$y = -2x$

14. $\{(1, -1), (2, -2), (3, -3), (4, -4)\}$ $y = -x$

Identify the independent and dependent variables in each situation.

15. Gardeners buy fertilizer according to the size of a lawn.
ind.: size of lawn; dep.: cost
16. The cost to gift wrap an order is \$3 plus \$1 per item wrapped.
ind.: items wrapped; dep.: cost

Identify the independent and dependent variables. Write an equation in function notation for each situation.

17. To rent a DVD, a customer must pay \$3.99 plus \$0.99 for every day that it is late.
ind.: days late; dep.: total cost; $f(x) = 3.99 + 0.99x$
18. Stephen charges \$25 for each lawn he mows.
ind.: lawns mowed; dep.: amount earned; $f(x) = 25x$
19. A car can travel 28 miles per gallon of gas.
ind.: gallons of gas; dep.: miles; $f(x) = 28x$

3-3 Exercises

Assignment Guide

Assign *Guided Practice* exercises as necessary.

If you finished Examples 1–3
Basic 13–19, 29
Average 13–19, 29
Advanced 13–19, 29

If you finished Examples 1–5
Basic 13–35
Average 13–36
Advanced 13–24, 29, 31–37

Homework Quick Check
Quickly check key concepts.
Exercises: 14, 16, 18, 20, 23, 24

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them. Exercises 12, 23–24, 28, 32, 37

Reason abstractly and quantitatively. Exercises 27, 36

Construct viable arguments and critique the reasoning of others. Exercises 29–31

Look for and make use of structure. Exercises 3–11, 13–22

Learn It Online
State Resources Online

MULTI-STEP TEST PREP **Exercise 32** involves determining relationships between two variables. This exercise prepares students for the Multi-Step Test Prep.

Answers

31. Possible answer: The cost of a hotel room is a function of how many nights you stay. The cost is the dependent variable because it depends on the number of nights; therefore, the number of nights is the independent variable.

Evaluate each function for the given input values.

20. For $f(x) = x^2 - 5$, find $f(x)$ when $x = 0$ and when $x = 3$. $f(0) = -5$; $f(3) = 4$

21. For $g(x) = x^2 + 6$, find $g(x)$ when $x = 1$ and when $x = 2$. $g(1) = 7$; $g(2) = 10$

22. For $f(x) = \frac{2}{3}x + 3$, find $f(x)$ when $x = 9$ and when $x = -3$. $f(9) = 9$; $f(-3) = 1$

23. A mail-order company charges \$5 per order plus \$2 per item in the order, up to a maximum of 4 items. Write a function to describe the situation. Find the reasonable domain and range for the function. $f(n) = 2n + 5$; D: {1, 2, 3, 4}; R: {\$7, \$9, \$11, \$13}

LINK Transportation

Air Force One refers to two specially configured Boeing 747-200B airplanes. The radio call sign when the president is aboard either aircraft or any Air Force aircraft is "Air Force One."

24. **Transportation** Air Force One can travel 630 miles per hour. Let h be the number of hours traveled. The function $d = 630h$ gives the distance d in miles that Air Force One travels in h hours.
   a. Identify the independent and dependent variables. Write $d = 630h$ using function notation. ind.: hours; dep.: distance; $f(h) = 630h$
   b. What are reasonable values for the domain and range in the situation described? D: $x \geq 0$; R: $y \geq 0$
   c. How far can Air Force One travel in 12 hours? 7560 mi

25. Complete the table for $g(z) = 2z - 5$.

z	1	2	3	4
$g(z)$	−3	−1	1	3

26. Complete the table for $h(x) = x^2 + x$.

x	0	1	2	3
$h(x)$	0	2	6	12

28a. D: $\{0 \leq g \leq 20\}$ R: $\{0 \leq d \leq 600\}$

27. **Estimation** For $f(x) = 3x + 5$, estimate the output when $x = -6.89$, $x = 1.01$, and $x = 4.67$. $f(-6.89) \approx -16$; $f(1.01) \approx 8$; $f(4.67) \approx 20$

28. **Transportation** A car can travel 30 miles on a gallon of gas and has a 20-gallon gas tank. Let g be the number of gallons of gas the car has in its tank. The function $d = 30g$ gives the distance d in miles that the car travels on g gallons.
   a. What are reasonable values for the domain and range in the situation described?
   b. How far can the car travel on 12 gallons of gas? 360 mi

29. Possible answer: There can be a maximum of 4 bowlers on one bowling lane, and each bowler needs to rent 2 shoes.

29. **Critical Thinking** Give an example of a real-life situation for which the reasonable domain consists of 1, 2, 3, and 4 and the reasonable range consists of 2, 4, 6, and 8.

30. Rashid must multiply \$150 by the number of months he saves for, not add the number of months to 150.

30. ERROR ANALYSIS Rashid saves \$150 each month. He wants to know how much he will have saved in 2 years. He writes the rule $s = m + 150$ to help him figure out how much he will save, where s is the amount saved and m is the number of months he saves. Explain why his rule is incorrect.

31. **Write About It** Give a real-life situation that can be described by a function. Identify the independent variable and the dependent variable.

MULTI-STEP TEST PREP

32. The table shows the volume v of water pumped into a pool after t hours.
   a. Determine a relationship between the time and the volume of water and write an equation. $v = 1250t$
   b. Identify the independent and dependent variables. ind.: time; dep.: volume
   c. If the pool holds 10,000 gallons, how long will it take to fill? 8h

Amount of Water in Pool	
Time (h)	Volume (gal)
0	0
1	1250
2	2500
3	3750
4	5000

(t), © Bettmann/CORBIS; (b), Sam Dudgeon/HMH

PRACTICE A

PRACTICE C

PRACTICE B

Determine a relationship between the *x*- and *y*-values. Write an equation.

1.

x	−4	−3	−2	−1
y	−1	0	1	2

$y = x + 3$

2. {(2, 3), (3, 5), (4, 7), (5, 9)}
$y = 2x - 1$

Identify the independent and dependent variables in each situation.

3. Ice cream sales increase when the temperature rises.
I: temperature; D: ice cream sales

4. Food for the catered party costs \$12.75 per person.
I: number of people; D: cost of food

Identify the independent and dependent variables. Write a rule in function notation for each situation.

5. Carson charges \$7 per hour for yard work.
I: number of hours; D: total charge; $f(h) = 7h$

6. Kay donates twice what Ed donates.
I: Ed's donation; D: Kay's donation; $f(d) = 2d$

Evaluate each function for the given input values.

7. For $f(x) = 5x + 1$, find $f(x)$ when $x = 2$ and when $x = 3$. 11; 16

8. For $g(x) = -4x$, find $g(x)$ when $x = -6$ and when $x = 2$. 24; −8

9. For $h(x) = x - 3$, find $h(x)$ when $x = 3$ and when $x = 1$. 0; −2

Complete the following.

10. An aerobics class is being offered once a week for 6 weeks. The registration fee is \$15 and the cost for each class attended is \$10. Write a function rule to describe the total cost of the class. Find a reasonable domain and range for the function.
$f(x) = 15 + 10x$; D: {0, 1, 2, 3, 4, 5, 6}; R: {15, 25, 35, 45, 55, 65, 75}

READING STRATEGIES

To read and write functions, you must understand what is meant by an *independent variable* and a *dependent variable*.

Carmen attends a school with a dress code. She went shopping and found acceptable shirts for \$15 each. Her mother has allowed her to buy up to 5 new shirts for the school year.

Independent variable: number of shirts
Dependent variable: total cost
The function that represents this relation is $f(x) = 15x$.
Reasonable domain (*x*-values): {1, 2, 3, 4, 5}
Reasonable range (*y*-values): {15, 30, 45, 60, 75}
When $x = 5$, $f(x) = 75$.
So, 5 shirts will cost \$75.

Answer the following based on the situation below.

Molly is making greeting cards for 5 of her friends. Each card is to have 4 ribbons.

1. Identify each as either the independent or the dependent variable.
"number of ribbons used" dependent
"number of cards made" independent
2. Write a function for the relation. $f(x) = 4x$
3. What is the reasonable domain for this function? D: {1, 2, 3, 4, 5}
4. What is the reasonable range for this function? R: {4, 8, 12, 16, 20}
5. How many ribbons will Molly use if she makes 3 cards? 12 ribbons
6. How many ribbons will Molly use if she makes 5 cards? 20 ribbons

RETEACH

Functions have dependent and independent variables. The dependent variable will always depend on the independent variable.

Rewrite each situation using the word *depends*. Then identify the dependent and the independent variables.

An employee who works longer hours will receive a larger amount in her paycheck.
Rewrite sentence:
The **amount of a paycheck** *depends* on the **number of hours worked**.
Dependent: amount of paycheck Independent: number of hours worked

A box with several books weighs more than a box with just a few books.
Rewrite sentence:
The **weight of a box** *depends* on the **number of books in the box**.
Dependent: weight of box Independent: number of books in box

Rewrite each sentence using the word *depends*. Then identify the dependent and the independent variables.

1. A very large animal will eat many pounds of food.
The amount of food eaten depends on the size of the animal.
Dependent: pounds of food
Independent: size of animal
2. The fire was very large, so many firefighters were there.
The number of firefighters depends on the size of the fire.
Dependent: number of firefighters
Independent: size of the fire
3. The temperature of the water on the heated stove rose each minute.
The temperature of the water depends on the time it has been on the stove.
Dependent: temperature of the water
Independent: time on the stove
4. The restaurant bill was low because only a few meals were ordered.
The amount of the restaurant bill depends on the number of meals ordered.
Dependent: amount of the restaurant bill
Independent: number of meals ordered

TEST PREP

33. Marsha buys x pens at \$0.70 per pen and one pencil for \$0.10. Which function gives the total amount Marsha spends?

Ⓐ $c(x) = 0.70x + 0.10x$
Ⓑ $c(x) = 0.70x + 1$
Ⓒ $c(x) = (0.70 + 0.10)x$
Ⓓ $c(x) = 0.70x + 0.10$

34. Belle is buying pizzas for her daughter's birthday party, using the prices in the table. Which equation best describes the relationship between the total cost c and the number of pizzas p?

Pizzas	Total Cost ($)
5	26.25
10	52.50
15	78.75

Ⓕ $c = 26.25p$
Ⓖ $c = 5.25p$
Ⓗ $c = p + 26.25$
Ⓙ $c = 6p - 3.75$

35. Gridded Response What is the value of $f(x) = 5 - \frac{1}{2}x$ when $x = 3$? **3.5**

CHALLENGE AND EXTEND

D: $0 \le x \le 100$; R: $32 \le x \le 212$

36. The formula to convert a temperature that is in degrees Celsius x to degrees Fahrenheit $f(x)$ is $f(x) = \frac{9}{5}x + 32$. What are reasonable values for the domain and range when you convert to Fahrenheit the temperature of water as it rises from 0° to 100° Celsius?

37. Math History In his studies of the motion of free-falling objects, Galileo Galilei found that regardless of its mass, an object will fall a distance d that is related to the square of its travel time t in seconds. The modern formula that describes free-fall motion is $d = \frac{1}{2}gt^2$, where g is the acceleration due to gravity and t is the length of time in seconds the object falls. Find the distance an object falls in 3 seconds. (*Hint*: Research to find acceleration due to gravity in meters per second squared.) **44.1 m**

PROBLEM SOLVING

Identify the independent and dependent variables. Write a rule in function notation for each situation.

1. Each state receives electoral votes based on the number of representatives it has in the House of Representatives.

Representatives	2	4	6	8
Electoral Votes	4	6	8	10

I: number of representatives; D: number of electoral votes; $f(r) = r + 2$

2. Terry has 30 pieces of gum and gives 2 pieces to each of his friends.

I: number of friends; D: pieces of gum Terry has left $f(x) = 30 - 2x$

3. Ronaldo is buying bacon that costs \$4.29 per pound.

I: pounds of bacon; D: total price; $f(b) = 4.29b$

4. A personal trainer charges \$50 for the first session and \$40 for every session thereafter.

I: number of sessions; D: total cost; $f(s) = 50 + 40(s - 1)$

International travel and business require the conversion of American dollars into foreign currency. During part of 2005, one American dollar was worth 6 Croatian Kuna. Select the best answer.

5. An American bank wishes to convert d dollars into kuna. Which function rule describes the situation?
A $f(d) = \frac{d}{6}$ C $f(d) = \frac{6}{d}$
B $f(d) = 6d$ D $f(d) = d + 6$

6. A Croatian company already has \$100,000 and is going to convert k kuna into dollars. Which function rule can be used to determine the total amount of American dollars this company will have?
F $f(x) = 100{,}000 + 6k$
G $f(x) = 100{,}000 + \frac{k}{6}$
H $f(x) = 100{,}000k + 6$
J $f(x) = 100{,}000 + \frac{6}{k}$

7. Macon has \$100 and is thinking about converting some of it into kuna. What is a reasonable range for this situation?
A $0 \le y \le 6$ C $0 \le y \le 100$
B $0 \le y \le 16.7$ D $0 \le y \le 600$

8. Robin converts x dollars into y kuna. Which expression is the independent variable in this situation?
F x H $6x$
G y J $6y$

9. Jakov converts n kuna into c dollars. Which expression is the dependent variable in this situation?
A n C $\frac{n}{6}$
B c D $\frac{c}{6}$

CHALLENGE

A Celsius thermometer shows a reading of 40 °C. While you are looking at this thermometer, the local weather report on the radio gives a temperature of 104 °F. When you change to another local station, you hear a report of 112 °F.

1. Use the formula $F = \frac{9}{5}C + 32$ to find the Fahrenheit temperature corresponding to 40 °C. **104°F**
2. Why do you suppose that you heard a different Fahrenheit temperature on one of the radio stations?
Answers may vary. Sample answer: the 112°F may have been a "heat index" reading.
3. Explain why $F = \frac{9}{5}C + 32$ represents a function.
For each value of C there is exactly 1 value for F.
4. Complete the following ordered pairs for (C, F) by using the formula $F = \frac{9}{5}C + 32$:
a. (25, **77**) b. (−10, **14**) c. (18, **64.4**) d. (**30**, 86)
5. In part **d** of Exercise 4, you solved for C given F. Use the appropriate properties of equality to solve $F = \frac{9}{5}C + 32$ for C in terms of F. **$C = \frac{5}{9}(F - 32)$**
6. Use the formula found in Exercise 5 to complete the following ordered pairs (C, F).
a. **15**, 59) b. (**25**, 77) c. (**−20**, −4) d. (**26$\frac{2}{3}$**, 80)
7. Is this formula for C in terms of F also a function? Why or why not?
Yes; for each value of F, there is exactly one value of C.
8. Functions that "undo" each other are called *inverse* functions. Start with 35 °C and use the first formula to find a value of F. Then use this value of F in the second formula to find a value of C. What happens?
The result is the original 35.
9. Find the temperature at which the Celsius and the Fahrenheit temperatures are equivalent.
There is no temperature equivalent in both systems.

TEST PREP DOCTOR

Exercise 33 states that one pencil is purchased for \$0.10. Therefore, the function must contain 0.10 as a constant (not multiplied by the variable). This is true only in choice **D.**

Journal

Have students give the meaning of $f(3) = 12$, and then create a function such that $f(3) = 12$.

ALTERNATIVE ASSESSMENT

Have students choose a math formula and write the formula in function notation, stating which is the dependent variable and which is the independent variable. Then have them evaluate their function for three appropriate domain values.

Power Presentations with PowerPoint®

3-3 Lesson Quiz

Identify the independent and dependent variables. Write an equation in function notation for each situation.

1. A buffet charges \$8.95 per person. ind.: number of people; dep.: cost; $f(p) = 8.95p$
2. A moving company charges \$130 for weekly truck rental plus \$1.50 per mile. ind.: miles; dep.: cost; $f(m) = 130 + 1.50m$

Evaluate each function for the given input values.

3. For $g(t) = \frac{1}{4}t - 3$, find $g(t)$ when $t = 20$ and when $t = -12$. $g(20) = 2$; $g(-12) = -6$
4. For $f(x) = 6x - 1$, find $f(x)$ when $x = 3.5$ and when $x = -5$. $f(3.5) = 20$; $f(-5) = -31$

Write a function to describe the situation. Find the reasonable domain and range for the function.

5. A theater can be rented for exactly 2, 3, or 4 hours. The cost is a \$100 deposit plus \$200 per hour. $f(h) = 100 + 200h$; D: {2, 3, 4}; R: {500, 700, 900}

Also available online

3-4 Organizer

Pacing: Traditional 1 day
Block $\frac{1}{2}$ day

Objectives: Graph functions given a limited domain.

Graph functions given a domain of all real numbers.

Online Edition
Tutorial Videos, Interactivity, TechKeys

Countdown

Power Presentations with PowerPoint®

Warm Up

Solve each equation for *y*.

1. $2x + y = 3$ $y = -2x + 3$
2. $-x + 3y = -6$ $y = \frac{1}{3}x - 2$
3. $4x - 2y = 8$ $y = 2x - 4$
4. Generate ordered pairs for $y = \frac{1}{2}x + 1$ using $x = -4$, -2, 0, 2, and 4. $(-4, -1)$, $(-2, 0)$, $(0, 1)$, $(2, 2)$, $(4, 3)$

Also available online

Math Humor

Psychology Teacher: Can anyone use "dysfunction" in a sentence?

Math Student: I can! "Dysfunction" is really hard to graph.

COMMON CORE

CC.9-12.F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.*
CC.9-12.F.IF.1 Understand that a function … assigns to each element of the domain exactly one element of the range. … The graph of *f* is the graph of the equation $y = f(x)$.
Also **CC.9-12.F.IF.2, CC.9-12.F.IF.7*, CC.9-12.A.REI.10**

Learn It Online
State Resources Online

COMMON CORE

3-4

Graphing Functions

CC.9-12.F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.* *Also* **CC.9-12.F.IF.1, CC.9-12.F.IF.2, CC.9-12.F.IF.7*, CC.9-12.A.REI.10**

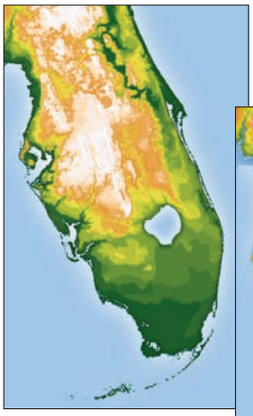
Objectives
Graph functions given a limited domain.

Graph functions given a domain of all real numbers.

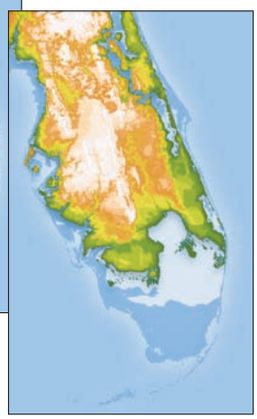
Who uses this?
Scientists can use a function to make conclusions about rising sea level.

Sea level is rising at an approximate rate of 2.5 millimeters per year. If this rate continues, the function $y = 2.5x$ can describe how many millimeters y sea level will rise in the next x years.

One way to understand functions such as the one above is to graph them. You can graph a function by finding ordered pairs that satisfy the function.



Current Florida coastline.



Possible Florida coastline in 2400 years.

EXAMPLE 1 Graphing Solutions Given a Domain

Graph each function for the given domain.

A $-x + 2y = 6$; D: $\{-4, -2, 0, 2\}$

Helpful Hint
Sometimes solving for *y* first makes it easier to substitute values of *x* and find an ordered pair.

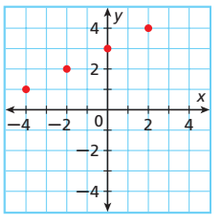
Step 1 Solve for *y* since you are given values of the domain, or *x*.

$-x + 2y = 6$
$+x \quad +x$ — *Add x to both sides.*
$2y = x + 6$
$\frac{2y}{2} = \frac{x+6}{2}$ — *Since y is multiplied by 2, divide both sides by 2.*
$y = \frac{x}{2} + \frac{6}{2}$ — *Rewrite $\frac{x+6}{2}$ as two separate fractions.*
$y = \frac{1}{2}x + 3$ — *Simplify.*

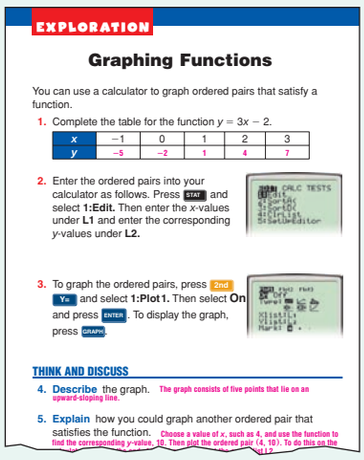
Step 2 Substitute the given values of the domain for *x* and find values of *y*.

x	$y = \frac{1}{2}x + 3$	(*x*, *y*)
−4	$y = \frac{1}{2}(-4) + 3 = 1$	(−4, 1)
−2	$y = \frac{1}{2}(-2) + 3 = 2$	(−2, 2)
0	$y = \frac{1}{2}(0) + 3 = 3$	(0, 3)
2	$y = \frac{1}{2}(2) + 3 = 4$	(2, 4)

Step 3 Graph the ordered pairs.



1 Introduce



EXPLORATION

Graphing Functions

You can use a calculator to graph ordered pairs that satisfy a function.

1. Complete the table for the function $y = 3x - 2$.

x	−1	0	1	2	3
y	−5	−2	1	4	7

2. Enter the ordered pairs into your calculator as follows. Press STAT and select **1:Edit.** Then enter the *x*-values under **L1** and enter the corresponding *y*-values under **L2.**

3. To graph the ordered pairs, press 2nd Y= and select **1:Plot 1.** Then select **On** and press ENTER. To display the graph, press GRAPH.

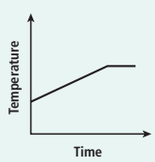
THINK AND DISCUSS

4. **Describe** the graph. The graph consists of five points that lie on an upward-sloping line.

5. **Explain** how you could graph another ordered pair that satisfies the function. Choose a value of *x*, such as 4, and use the function to find the corresponding *y*-value, 10. Then plot the ordered pair (4, 10). To do this on the

Motivate

Display the following graph:

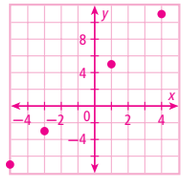


1. Describe the graph.
Possible answer: The temperature increases and then remains constant.

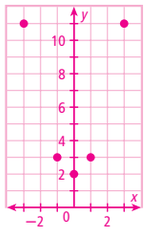
2. Can you determine what the temperature will be after 3 hours? If so, what is it? cannot be determined

Explorations and answers are provided in your online resources.

1a.



b.



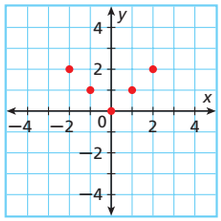
Graph each function for the given domain.

B $f(x) = |x|$; D: $\{-2, -1, 0, 1, 2\}$

Step 1 Use the given values of the domain to find values of $f(x)$.

x	$f(x) = \lvert x \rvert$	$(x, f(x))$
-2	$f(x) = \lvert -2 \rvert = 2$	$(-2, 2)$
-1	$f(x) = \lvert -1 \rvert = 1$	$(-1, 1)$
0	$f(x) = \lvert 0 \rvert = 0$	$(0, 0)$
1	$f(x) = \lvert 1 \rvert = 1$	$(1, 1)$
2	$f(x) = \lvert 2 \rvert = 2$	$(2, 2)$

Step 2 Graph the ordered pairs.



CHECK IT OUT! Graph each function for the given domain.

1a. $-2x + y = 3$; D: $\{-5, -3, 1, 4\}$

1b. $f(x) = x^2 + 2$; D: $\{-3, -1, 0, 1, 3\}$

If the domain of a function is all real numbers, any number can be used as an input value. This process will produce an infinite number of ordered pairs that satisfy the function. Therefore, arrowheads are drawn at both "ends" of a smooth line or curve to represent the infinite number of ordered pairs. If a domain is not given, assume that the domain is all real numbers.

Know it! Note

Graphing Functions Using a Domain of All Real Numbers	
Step 1	Use the function to generate ordered pairs by choosing several values for x.
Step 2	Plot enough points to see a pattern for the graph.
Step 3	Connect the points with a line or smooth curve.

EXAMPLE 2 Graphing Functions

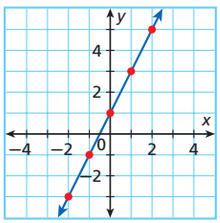
Graph each function.

A $2x + 1 = y$

Step 1 Choose several values of x and generate ordered pairs.

Step 2 Plot enough points to see a pattern.

x	$2x + 1 = y$	(x, y)
-3	$2(-3) + 1 = -5$	$(-3, -5)$
-2	$2(-2) + 1 = -3$	$(-2, -3)$
-1	$2(-1) + 1 = -1$	$(-1, -1)$
0	$2(0) + 1 = 1$	$(0, 1)$
1	$2(1) + 1 = 3$	$(1, 3)$
2	$2(2) + 1 = 5$	$(2, 5)$
3	$2(3) + 1 = 7$	$(3, 7)$



Step 3 The ordered pairs appear to form a line. Draw a line through all the points to show all the ordered pairs that satisfy the function. Draw arrowheads on both "ends" of the line.

Helpful Hint

When choosing values of x, be sure to choose both positive and negative values. You may not need to graph all the points to see the pattern.

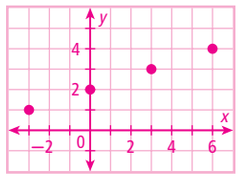
Power Presentations with PowerPoint®

Additional Examples

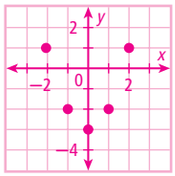
Example 1

Graph each function for the given domain.

A. $x - 3y = -6$; D: $\{-3, 0, 3, 6\}$



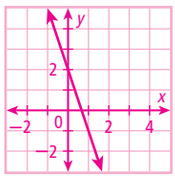
B. $f(x) = x^2 - 3$; D: $\{-2, -1, 0, 1, 2\}$



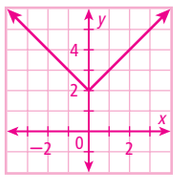
Example 2

Graph each function.

A. $-3x + 2 = y$



B. $g(x) = |x| + 2$



Also available online

2 Teach

Guided Instruction

Be sure students can solve for y and graph ordered pairs before beginning this lesson. Point out that graphing a function with given values for the domain will yield distinct points, while graphing a function using all real numbers yields a continuous graph. This is shown by arrows on the graphs of continuous functions. Stress the importance of choosing several points (positive, negative, and zero values) when graphing to determine the pattern.

Reaching All Learners

Through Modeling

The function $y = 2.5x$ describes the mass in grams of a stack of x pennies minted after 1982. Have students create a table of values and graph the function.

Obtain a scale (in grams) and several pennies (post-1982). Ask students to use their graph to predict the mass of a given number of pennies.

Measure the mass of the pennies to check the answers given. Repeat with a different number of pennies.

Questioning Strategies

EXAMPLE 1

- What is a benefit to solving for y when graphing a function?
- When creating the table, what did you notice about the y-values relative to the x-values?

EXAMPLE 2

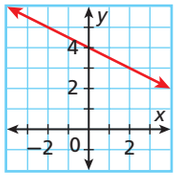
- What is the difference between this example and **Example 1**?

Power Presentations with PowerPoint®

Additional Examples

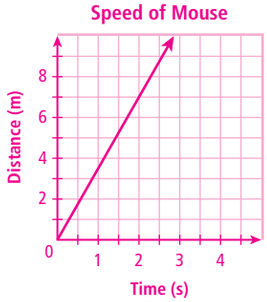
Example 3

Use a graph of the function $f(x) = -\frac{1}{2}x + 4$ to find the value of $f(x)$ when $x = -4$. Check your answer. $f(-4) = 6$



Example 4

A mouse can run 3.5 meters per second. The function $y = 3.5x$ describes the distance in meters the mouse can run in x seconds. Graph the function. Use the graph to estimate how many meters a mouse can run in 2.5 seconds. about 8.75 meters



Also available online

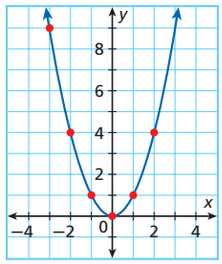
Graph each function.

B $y = x^2$

Step 1 Choose several values of x and generate ordered pairs.

Step 2 Plot enough points to see a pattern.

x	$y = x^2$	(x, y)
−3	$y = (-3)^2 = 9$	(−3, 9)
−2	$y = (-2)^2 = 4$	(−2, 4)
−1	$y = (-1)^2 = 1$	(−1, 1)
0	$y = (0)^2 = 0$	(0, 0)
1	$y = (1)^2 = 1$	(1, 1)
2	$y = (2)^2 = 4$	(2, 4)



Step 3 The ordered pairs appear to form an almost U-shaped graph. Draw a smooth curve through the points to show all the ordered pairs that satisfy the function. Draw arrowheads on the "ends" of the curve.

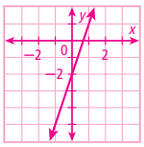
Check If the graph is correct, any point on it will satisfy the function. Choose an ordered pair on the graph that was not in your table, such as (3, 9). Check whether it satisfies $y = x^2$.

$y = x^2$	
9	3^2
9	9 ✓

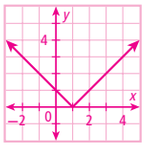
Substitute the values for x and y into the function. Simplify.

The ordered pair (3, 9) satisfies the function.

2a.



2b.



CHECK IT OUT! **Graph each function.**

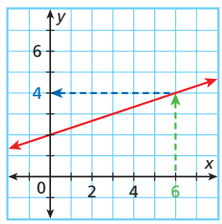
2a. $f(x) = 3x - 2$

2b. $y = |x - 1|$

EXAMPLE 3 Finding Values Using Graphs

Use a graph of the function $f(x) = \frac{1}{3}x + 2$ to find the value of $f(x)$ when $x = 6$. Check your answer.

Locate 6 on the x-axis. Move up to the graph of the function. Then move left to the y-axis to find the corresponding value of y.



$f(x) = 4$

Check Use substitution.

$f(x) = \frac{1}{3}x + 2$	
4	$\frac{1}{3}(6) + 2$
4	$2 + 2$
4	4 ✓

Substitute the values for x and y into the function.

Simplify.

The ordered pair (4, 6) satisfies the function.

Writing Math

"The value of y is 4 when $x = 6$" can also be written as $f(6) = 4$.

CHECK IT OUT! **3.** Use the graph above to find the value of x when $f(x) = 3$. Check your answer. 3

MATHEMATICAL PRACTICES Questioning Strategies

EXAMPLE 3

- Can the graph be used to find the value of y if x is a fractional number? Explain.

EXAMPLE 4

- If you wanted an exact answer, what could you do?

Reaching All Learners *Through Multiple Representations*

The period of revolution for Mars (compared to Earth) is approximately 1.88 Earth years. Have students find their age in Mars years by using the function $y = \frac{1}{1.88}x$ or $y = 0.5319x$, where y = age in Mars years and x = age in Earth years.

Have students create a table of values and graph the function. Use the graph to estimate various ages in Mars years.

Recall that in real-world situations you may have to limit the domain to make answers reasonable. For example, quantities such as time, distance, and number of people can be represented using only nonnegative values. When both the domain and the range are limited to nonnegative values, the function is graphed only in Quadrant I.

EXAMPLE 4 *Problem-Solving Application*

The function $y = 2.5x$ describes how many millimeters sea level y rises in x years. Graph the function. Use the graph to estimate how many millimeters sea level will rise in 3.5 years.

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them.

1 Understand the Problem

The **answer** is a graph that can be used to find the value of y when x is 3.5.

List the important information:

- The function $y = 2.5x$ describes how many millimeters sea level rises.

2 Make a Plan

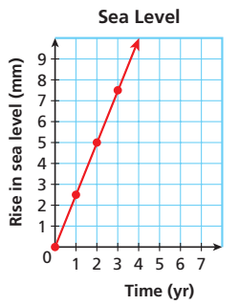
Think: What values should I use to graph this function? Both, the number of years sea level has risen and the distance sea level rises, cannot be negative. Use only nonnegative values for both the domain and the range. The function will be graphed in Quadrant I.

3 Solve

Choose several nonnegative values of x to find values of y. Then graph the ordered pairs.

x	$y = 2.5x$	(x, y)
0	$y = 2.5(0) = 0$	(0, 0)
1	$y = 2.5(1) = 2.5$	(1, 2.5)
2	$y = 2.5(2) = 5$	(2, 5)
3	$y = 2.5(3) = 7.5$	(3, 7.5)
4	$y = 2.5(4) = 10$	(4, 10)

Draw a line through the points to show all the ordered pairs that satisfy this function.

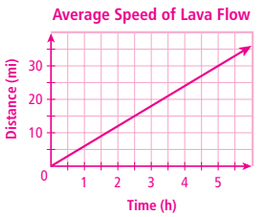


Use the graph to estimate the y-value when x is 3.5.
Sea level will rise about 8.75 millimeters in 3.5 years.

4 Look Back

As the number of years increases, sea level also increases, so the graph is reasonable. When x is between 3 and 4, y is between 7.5 and 10. Since 3.5 is between 3 and 4, it is reasonable to estimate y to be 8.75 when x is 3.5.

4. Possible answer: about 32.5 mi



CHECK IT OUT!

4. The fastest recorded Hawaiian lava flow moved at an average speed of 6 miles per hour. The function $y = 6x$ describes the distance y the lava moved on average in x hours. Graph the function. Use the graph to estimate how many miles the lava moved after 5.5 hours.

COMMON ERROR ALERT

In **Example 4,** students may simply evaluate the function for $x = 3.5$ rather than using the graph. Tell students that equations will become more complicated later, and they must learn to read values from a graph.

Teaching Tip **Inclusion** When graphing a function, suggest that students choose simple values for x, such as 0, 1, and 2. They should also be sure to choose negative values when the domain is all real numbers.

Teaching Tip **Critical Thinking** Discuss with students what negative values of x could represent. For example, if x represents time in years, lead them to see that an x-value of -2 could represent 2 years ago.

3 Close

Summarize

Have students name the steps for graphing a function. solve for y, choose points for x, generate ordered pairs, graph Remind students that the points are not connected when the domain is a finite set of values. The points are connected when the domain is the set of all real numbers. When not otherwise stated, the domain will be the set of real numbers.

ONGOING ASSESSMENT and INTERVENTION

***Diagnose Before* the Lesson**
Warm Up, TE

***Monitor During* the Lesson**
Check It Out! Exercises, SE
Questioning Strategies, TE

***Assess After* the Lesson**
Lesson Quiz, TE
Alternative Assessment, TE

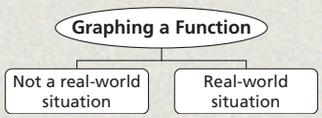
Answers to *Think and Discuss*

1. Possible answer: Graph the function by plotting points and connecting them with a line or smooth curve. Then use the graph to determine all possible values of y (the range).
2. Locate the given value on the x-axis. Move up or down to the graph, and then left or right to the y-axis to find the y-value.
3. See Additional Answers.

THINK AND DISCUSS

MATHEMATICAL PRACTICES

1. How do you find the range of a function if the domain is all real numbers?
2. Explain how to use a graph to find the value of a function for a given value of x.
3. **GET ORGANIZED** Copy and complete the graphic organizer. Explain how to graph a function for each situation.



3-4 Exercises

Assignment Guide

Assign *Guided Practice* exercises as necessary.

If you finished Examples **1–2**
Basic 13–24, 40–43
Average 13–24, 45–47, 62
Advanced 13–24, 45–47, 62

If you finished Examples **1–4**
Basic 13–55, 58–61
Average 13–62
Advanced 13–43, 45–63

Homework Quick Check
Quickly check key concepts.
Exercises: 14, 22, 26, 27, 34, 50

Teaching Tip **Math Background**
For **Exercise 10,** remind students that $-x^2$ is the same as $-(x^2)$.

Answers

1–10, 12–24. See Additional Answers.

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them. Exercises 12, 27, 55, 63

Reason abstractly and quantitatively. Exercise 56

Construct viable arguments and critique the reasoning of others. Exercises 44, 53, 57

Look for and make use of structure. Exercises 1–11, 13–26, 28–43, 45–53, 62

Learn It Online
State Resources Online

3-4 Exercises

Learn It Online
Homework Help Online
Parent Resources Online

GUIDED PRACTICE

SEE EXAMPLE 1 — Graph each function for the given domain.

1. $3x - y = 1$; D: $\{-3, -1, 0, 4\}$
2. $f(x) = -|x|$; D: $\{-5, -3, 0, 3, 5\}$
3. $f(x) = x + 4$; D: $\{-5, -3, 0, 4\}$
4. $y = x^2 - 1$; D: $\{-3, -1, 0, 1, 3\}$

SEE EXAMPLE 2 — Graph each function.

5. $f(x) = 6x + 4$
6. $y = \frac{1}{2}x + 4$
7. $x + y = 0$
8. $y = |x| - 4$
9. $f(x) = 2x^2 - 7$
10. $y = -x^2 + 5$

SEE EXAMPLE 3

11. Use a graph of the function $f(x) = \frac{1}{2}x - 2$ to find the value of y when $x = 2$. Check your answer. $y = -1$

SEE EXAMPLE 4

12. **Oceanography** The floor of the Atlantic Ocean is spreading at an average rate of 1 inch per year. The function $y = x$ describes the number of inches y the ocean floor spreads in x years. Graph the function. Use the graph to estimate the number of inches the ocean floor will spread in $10\frac{1}{2}$ years.

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
13–16	1
17–24	2
25–26	3
27	4

Extra Practice
See Extra Practice for more Skills Practice and Applications Practice exercises.

Graph each function for the given domain.

13. $2x + y = 4$; D: $\{-3, -1, 4, 7\}$
14. $y = |x| - 1$; D: $\{-4, -2, 0, 2, 4\}$
15. $f(x) = -7x$; D: $\{-2, -1, 0, 1\}$
16. $y = (x + 1)^2$; D: $\{-2, -1, 0, 1, 2\}$

Graph each function.

17. $y = -3x + 5$
18. $f(x) = 3x$
19. $x + y = 8$
20. $f(x) = 2x + 2$
21. $y = -|x| + 10$
22. $f(x) = -5 + x^2$
23. $y = |x + 1| + 1$
24. $y = (x - 2)^2 - 1$
25. Use a graph of the function $f(x) = -2x - 3$ to find the value of y when $x = -4$. Check your answer. $y = 5$
26. Use a graph of the function $f(x) = \frac{1}{3}x + 1$ to find the value of y when $x = 6$. Check your answer. $y = 3$

READING STRATEGIES

Whether graphing a function with a limited domain, or a domain of all real numbers, the procedure is almost the same. The steps of the procedure are listed below and shown in the following example.

Step 1: Create a three-column table headed "x", "$f(x)$", and "$(x, f(x))$".
Step 2: Write the given or chosen domain values in the "x" column.
Step 3: Substitute the domain values into the function and evaluate for $f(x)$.
Step 4: Write corresponding x- and $f(x)$-values as ordered pairs.
Step 5: Plot the ordered pairs on a coordinate grid.
Step 6: If the domain is all real numbers, connect the points with a smooth line/curve and put arrows on both ends.

Graph the function $f(x) = |x - 2|$.

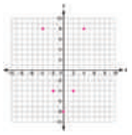
Step1

x	$f(x)$	$(x, f(x))$	
6	$\|6 - 2\| = \|4\| = 4$	(6, 4)	Step5
4	$\|4 - 2\| = \|2\| = 2$	(4, 2)	
2	$\|2 - 2\| = \|0\| = 0$	(2, 0)	Step6
0	$\|0 - 2\| = \|-2\| = 2$	(0, 2)	
−2	$\|-2 - 2\| = \|-4\| = 4$	(−2, 4)	

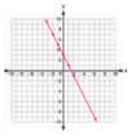
Step2 Step3 Step4

Graph each function using the procedure shown above.

1. $f(x) = x^2 - 8$; D: $\{-4, -2, 0, 2, 4\}$



2. $f(x) = -2x + 3$

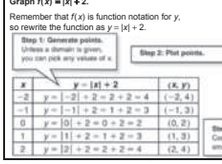


RETEACH

There are three steps to graphing a function.

Graph $f(x) = |x| + 2$.

Remember that $f(x)$ is function notation for y, so rewrite the function as $y = |x| + 2$.



x	$y = \|x\| + 2$	(x, y)
−2	$y = \|-2\| + 2 = 2 + 2 = 4$	(−2, 4)
−1	$y = \|-1\| + 2 = 1 + 2 = 3$	(−1, 3)
0	$y = \|0\| + 2 = 0 + 2 = 2$	(0, 2)
1	$y = \|1\| + 2 = 1 + 2 = 3$	(1, 3)
2	$y = \|2\| + 2 = 2 + 2 = 4$	(2, 4)

Graph each function.

1. $y = (x + 2)^2$

x	$y = (x + 2)^2$	(x, y)
−4	$y = (-4 + 2)^2 = (-2)^2 = 4$	(−4, 4)
−3	$y = (-3 + 2)^2 = (-1)^2 = 1$	(−3, 1)
−2	$y = (-2 + 2)^2 = (0)^2 = 0$	(−2, 0)
−1	$y = (-1+2)^2 = (1)^2 = 1$	(−1, 1)
0	$y = (0+2)^2 = (2)^2 = 4$	(0, 4)

2. $f(x) = \frac{1}{2}x - 3$

x	$y = \frac{1}{2}x - 3$	(x, y)
−4	$y = \frac{1}{2}(-4) - 3 = -2 - 3 = -5$	(−4, −5)
−2	$y = \frac{1}{2}(-2) - 3 = -1 - 3 = -4$	(−2, −4)
0	$y = \frac{1}{2}(0) - 3 = 0 - 3 = -3$	(0, −3)
2	$y = \frac{1}{2}(2) - 3 = 1 - 3 = -2$	(2, −2)
4	$y = \frac{1}{2}(4) - 3 = 2 - 3 = -1$	(4, −1)

27. Transportation An electric motor scooter can travel at 0.25 miles per minute. The function $y = 0.25x$ describes the number of miles y the scooter can travel in x minutes. Graph the function. Use the graph to estimate the number of miles an electric motor scooter travels in 15 minutes.

Graph each function.

28. $f(x) = x - 1$

29. $12 - x - 2y = 0$

30. $3x - y = 13$

31. $y = x^2 - 2$

32. $x^2 - y = -4$

33. $2x^2 = f(x)$

34. $f(x) = |2x| - 2$

35. $y = |-x|$

36. $-|2x + 1| = y$

37. Find the value of x so that $(x, 12)$ satisfies $y = 4x + 8$. **1**

38. Find the value of x so that $(x, 6)$ satisfies $y = -x - 4$. **−10**

39. Find the value of y so that $(-2, y)$ satisfies $y = -2x^2$. **−8**

For each function, determine whether the given points are on the graph.

40. $y = 7x - 2$; $(1, 5)$ and $(2, 10)$ **yes; no**

41. $y = |x| + 2$; $(3, 5)$ and $(-1, 3)$ **yes; yes**

42. $y = x^2$; $(1, 1)$ and $(-3, -9)$ **yes; no**

43. $y = \frac{1}{4}x - 2$; $\left(1, -\frac{3}{4}\right)$ and $(4, -1)$ **no; yes**

44. ERROR ANALYSIS Student A says that $(3, 2)$ is on the graph of $y = 4x - 5$, but student B says that it is not. Who is incorrect? Explain the error.

Student A; student A substituted the coordinates of the ordered pair incorrectly, and student B substituted them correctly.

A

$y = 4x - 5$	
3	$4(2) - 5$
3	$8 - 5$
3	3 ✓

B

$y = 4x - 5$	
2	$4(3) - 5$
2	$12 - 5$
2	7 ✗

Determine whether $(0, -7)$, $\left(-6, -\frac{5}{3}\right)$, and $(-2, -3)$ lie on the graph of each function.

45. $x + 3y = -11$ **no; yes; yes**

46. $y + |x| = -1$ **no; no; yes**

47. $x^2 - y = 7$ **yes; no; yes**

For each function, find three ordered pairs that lie on the graph of the function.

48. $-6 = 3x + 2y$

49. $y = 1.1x + 2$

50. $y = \frac{4}{5}x$

51. $y = 3x - 1$

52. $y = |x| + 6$

53. $y = x^2 - 5$

54. Critical Thinking Graph the functions $y = |x|$ and $y = -|x|$. Describe how they are alike. How are they different?

54. Possible answer: The graphs are alike because they are both V shaped. They are different because the graph of $y = |x|$ opens upward and the graph of $y = -|x|$ opens downward.

Sam Dudgeon/HMH

55. A pool containing 10,000 gallons of water is being drained. Every hour, the volume of the water in the pool decreases by 1500 gallons.

a. Write an equation to describe the volume v of water in the pool after h hours.

b. How much water is in the pool after 1 hour?

c. Create a table of values showing the volume of the water in gallons in the pool as a function of the time in hours and graph the function.

MULTI-STEP TEST PREP **Exercise 55** involves writing an equation and creating a table of values. This exercise prepares students for the Multi-Step Test Prep.

Answers

27. about 3.75 mi

Electric Scooter Mileage

28–36. See Additional Answers.

48–53. Possible answers given.

48. (0, −3), (−2, 0), (2, −6)

49. (0, 2), (1, 3.1), (−1, 0.9)

50. (0, 0), $\left(1, \frac{4}{5}\right)$, $\left(-1, -\frac{4}{5}\right)$

51. (0, −1), (1, 2), (−2, −7)

52. (0, 6), (1, 7), (2, 8)

53. (0, −5), (1, −4), (−1, −4)

55. See Additional Answers.

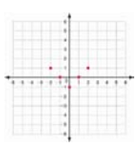
PRACTICE A

PRACTICE C

PRACTICE B

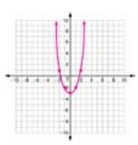
Graph the function for the given domain.

1. $y = |x| - 1$; D: {−1, 0, 1, 2, 3}



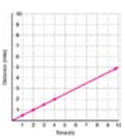
Graph the function.

2. $f(x) = x^2 - 3$



3. One of the slowest fish is the blenny fish. The function $y = 0.5x$ describes how many miles y the fish swims in x hours. Graph the function. Use the graph to estimate the number of miles the fish swims in 3.5 hours.

about 1.75 miles



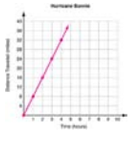
PROBLEM SOLVING

In 1998, Hurricane Bonnie approached the United States at a speed of 8 miles per hour. The function $y = 8x$ describes how many miles y Hurricane Bonnie traveled in x hours.

1. Complete the table by generating ordered pairs.

x	$y = 8x$	(x, y)
0	0	(0, 0)
1	8	(1, 8)
2	16	(2, 16)
3	24	(3, 24)
4	32	(4, 32)

2. Graph the function $y = 8x$.

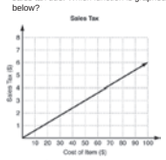


3. Use the graph to estimate how far Hurricane Bonnie traveled in 3.5 hours.

28 miles

Select the correct answer.

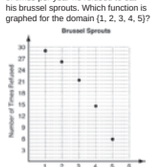
4. The graph below shows the relation between the cost of an item and the sales tax due. Which function is graphed below?



A $y = \frac{6}{x}$ C $y = \frac{x}{6}$

B $y = 0.06x$ D $y = 6x$

5. The graph below shows the relation between Jeremy's age and the number of times per year he refused to eat his brussel sprouts. Which function is graphed for the domain {1, 2, 3, 4, 5}?



F $y = 30 - x$ H $y = 30 - x^2$

G $y = x + 28$ J $y = 29x$

CHALLENGE

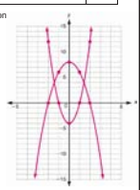
When the output of one function is used as the input of another, you have a **composition of functions.** The notation $f(g(x))$ means you input x into function g, and then use that output as the input for function f.

For 1–5, use the functions $f(x) = x^2 - 4$ and $g(x) = -2x$.

1. Use this table to find $f(g(x))$ for each x-value.

x	$g(x) = -2x$	$g(x)$	$f(x) = x^2 - 4$	$f(g(x))$
−2	$g(-2) = -2(-2) = 4$	4	$f(4) = 4^2 - 4 = 16 - 4 = 12$	12
	$g(-1) = -2(-1) = 2$	2	$f(2) = 2^2 - 4 = 4 - 4 = 0$	0
	$g(0) = -2(0) = 0$	0	$f(0) = 0^2 - 4 = 0 - 4 = -4$	−4
	$g(1) = -2(1) = -2$	−2	$f(-2) = (-2)^2 - 4 = 4 - 4 = 0$	0
	$g(2) = -2(2) = -4$	−4	$f(-4) = (-4)^2 - 4 = 16 - 4 = 12$	12

2. Use the first and last columns of the table to graph the function $y = f(g(x))$. Connect the points with a smooth curve.



Here is the algebra to find a "one-step" rule for $y = f(g(x))$.

$y = f(g(x))$ *Write the composite function.*

$y = f(-2x)$ *Because $g(x) = -2x$, substitute $-2x$ for $g(x)$.*

$y = (-2x)^2 - 4$ *Substitute $-2x$ for x in $f(x) = x^2 - 4$.*

$y = 4x^2 - 4$ *Simplify the power: $(-2x)^2 = (-2x)(-2x) = (-2)(-2)(x)(x) = 4x^2$.*

3. Use a similar process to find a one-step rule for $y = g(f(x))$.

$y = -2x^2 + 8$

4. Complete the table below for $y = g(f(x))$. Then graph the function. Use the same coordinate plane at right.

x	$y = g(f(x)) = -2x^2 + 8$	$(x, g(f(x)))$
−2	$g(f(-2)) = -2(-2)^2 + 8 = -8 + 8 = 0$	(−2, 0)
−1	$g(f(-1)) = -2(-1)^2 + 8 = -2 + 8 = 6$	(−1, 6)
0	$g(f(0)) = -2(0)^2 + 8 = 0 + 8 = 8$	(0, 8)
1	$g(f(1)) = -2(1)^2 + 8 = -2 + 8 = 6$	(1, 6)
2	$g(f(2)) = -2(2)^2 + 8 = -8 + 8 = 0$	(2, 0)

5. Is $f(g(x))$ the same as $g(f(x))$? No

TEST PREP DOCTOR In **Exercise 61,** if statement **I** is true, then statement **III** must also be true. Students who chose **H** may have confused the *x*- and *y*-axes.

Journal

Use $y = x^2 + 1$ to explain how you choose points when graphing a function. Graph the function.

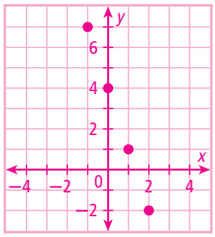
ALTERNATIVE ASSESSMENT

Create a function, write a table of four ordered pairs, and graph the function. Write an ordered pair for a point on the graph that is not included in the table.

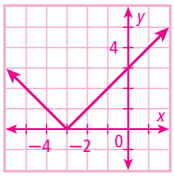
Power Presentations with PowerPoint®

3-4 Lesson Quiz

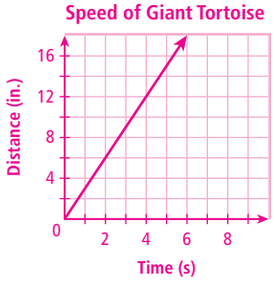
1. Graph the function for the given domain.
$3x + y = 4$; D: $\{-1, 0, 1, 2\}$



2. Graph the function $y = |x + 3|$.

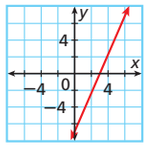


3. The function $y = 3x$ describes the distance (in inches) a giant tortoise walks in x seconds. Graph the function. Use the graph to estimate how many inches the tortoise will walk in 5.5 seconds.
about 16.5 in.



Also available online

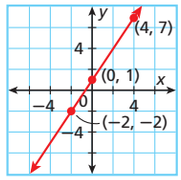
56. Estimation Use the graph to estimate the value of y when $x = 2.117$. $y \approx -2$



57. Write About It Why is a graph a convenient way to show the ordered pairs that satisfy a function?
A graph can represent all ordered pairs that satisfy a function, even when there are infinitely many.

TEST PREP

58. Which function is graphed?



(A) $2y - 3x = 2$ (C) $y = 2x - 1$
(B) $5x + y = 1$ (D) $y = 5x + 8$

59. Which ordered pair is NOT on the graph of $y = 4 - |x|$?
(F) $(0, 4)$ (H) $(-1, 3)$
(G) $(4, 0)$ (J) $(3, -1)$

60. Which function has $(3, 2)$ on its graph?
(A) $2x - 3y = 12$ (C) $y = -\frac{2}{3}x + 4$
(B) $-2x - 3y = 12$ (D) $y = -\frac{3}{2}x + 4$

61. Which statement(s) is true about the function $y = x^2 + 1$?
I. All points on the graph are above the origin.
II. All ordered pairs have positive *x*-values.
III. All ordered pairs have positive *y*-values.
(F) I Only (G) II Only (H) I and II (J) I and III

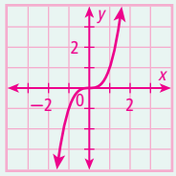
CHALLENGE AND EXTEND

62. Graph the function $y = x^3$. Make sure you have enough ordered pairs to see the shape of the graph.

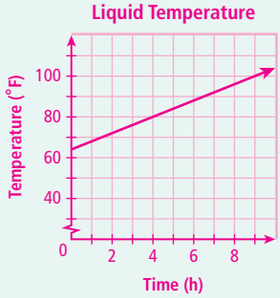
63. The temperature of a liquid that started at 64 °F is increasing by 4 °F per hour. Write a function that describes the temperature of the liquid over time. Graph the function to show the temperatures over the first 10 hours.

Answers

62.



63. $y = 4x + 64$



3-4 Technology LAB

Connect Function Rules, Tables, and Graphs

You can use a graphing calculator to understand the connections among function rules, tables, and graphs.

Use with Graphing Functions

CC.9-12.F.IF.1 Understand that a function … assigns to each element of the domain exactly one element of the range. … The graph of *f* is the graph of the equation $y = f(x)$.

Learn It Online
Lab Resources Online

MATHEMATICAL PRACTICES **Use appropriate tools strategically.**

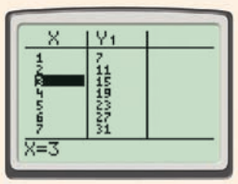
Activity

Make a table of values for the function $f(x) = 4x + 3$. Then graph the function.

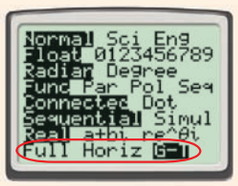
1. Press Y= and enter the function rule **4x + 3**.

2. Press 2nd TBLSET WINDOW. Make sure **Indpnt: Auto** and **Depend: Auto** are selected.

3. To view the table, press 2nd TABLE GRAPH. The *x*-values and the corresponding *y*-values appear in table form. Use the up and down arrow keys to scroll through the table.

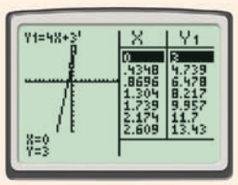


4. To view the table with the graph, press MODE and select **G-T** view. Press ENTER. Be sure to use the standard window.



5. Press TRACE to see both the graph and a table of values.

6. Press the left arrow key several times to move the cursor. Notice that the point on the graph and the values in the table correspond.



Try This

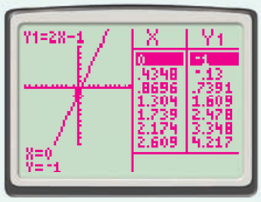
Make a table of values for each function. Then graph the function.

1. $f(x) = 2x - 1$
2. $f(x) = 1.5x$
3. $f(x) = \frac{1}{2}x + 2$
4. Explain the relationship between a function, its table of values, and the graph of the function.

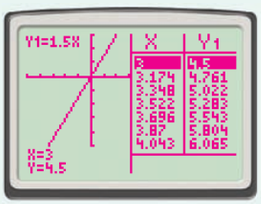
The table of values and graph of the function show ordered pairs that satisfy the function.

Answers to *Try This*

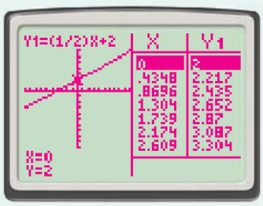
1.



2.



3.



Teaching Tip **Technology** Point out to students that **TblStart** defines the *x*-value at which the table values start and that **ΔTbl** defines the increment at which the *x*-values increase. Have students change these values if they are not already set to 1.

Technology LAB Organizer

Use with the lesson *Graphing Functions*

Pacing:
Traditional $\frac{1}{2}$ day
Block $\frac{1}{4}$ day

Objective: Use a graphing calculator to make the connections among equations, tables, and graphs.

Materials: Graphing calculator

INTERACTIVE **Online Edition**
Graphing Calculator

Countdown

Resources

Technology Lab Activities
Lab Recording Sheet

Teach

Discuss

Point out to students that all pairs of decimal values in the table are also ordered pairs.

Close

Key Concept

Equations, tables, and graphs can represent the same ordered pairs in different formats.

Assessment

Journal Create a function rule. Make a table of function values and list three ordered pairs. Then graph the ordered pairs and sketch the line or curve.

MATHEMATICAL PRACTICES

Use appropriate tools strategically.

COMMON CORE

CC.9-12.F.IF.1 Understand that a function … assigns to each element of the domain exactly one element of the range. … The graph of *f* is the graph of the equation $y = f(x)$.

Learn It Online
State Resources Online

MULTI-STEP TEST PREP

SECTION 3A

MULTI-STEP TEST PREP

Organizer

Objective: Assess students' ability to apply concepts and skills in a real-world format.

INTERACTIVE **Online Edition**

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them.

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them.

Function Concepts

Down the Drain The graph shows the relationship between the number of hours that have passed since a pool began to drain and the amount of water in the pool.

1. Describe in words the relationship between the amount of water in the pool and the number of hours that have passed since the pool began to drain.

2. What are the domain and range for the graph?

2. $0 \le t \le 7$; $0 \le v \le 1400$

3. Use the graph to determine how much water is in the pool after 3 hours. How much water is in the pool after $4\frac{1}{2}$ hours? 800 gal; 500 gal

4. Copy and complete the table.

Draining Pool	
Time (h)	Volume (gal)
0	1400
1	1200
2	1000
3	800
4	600
5	400
6	200
7	0

5. Write an equation to describe the relationship between the volume v and the time t. Use the equation to find how much water is in the pool after 5.2 hours.
$v = 1400 - 200t$; 360 gal

(br), Big Cheese Photo/Alamy Photos; (tl), Sam Dudgeon/HMH

1. Possible answer: The pool begins with 1400 gallons. As time goes on, the volume of water decreases by 200 gallons per hour.

INTERVENTION

Scaffolding Questions

1. When draining the pool, what happens to the volume of the water? As the pool drains, the volume of water decreases.
2. Does the volume depend on time, or does time depend on the volume? Volume depends on time.
3. Which axis is labeled with hours? horizontal Which axis is labeled with the amount of water? vertical
4. What volume does the graph show at 1 hour? 1200 at 2 hours? 1000
5. What is the rate at which the water is decreasing per hour as the pool drains? The water is decreasing at 200 gallons per hour.

Extension

Write a function that describes the relationship between the volume V and the time t for a pool that initially contains 800 gallons and drains at 100 gallons per hour.
$V(t) = 800 - 100t$

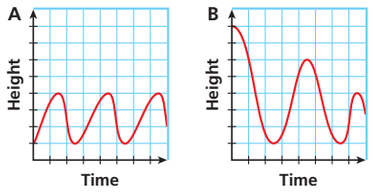
Learn It Online
State Resources Online

Quiz for Lessons 3-1 Through 3-4

3-1 Graphing Relationships

Choose the graph that best represents each situation.

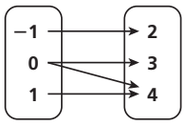
1. A person bungee jumps from a high platform. **B**
2. A person jumps on a trampoline in a steady motion. **A**
3. Xander takes a quiz worth 100 points. Each question is worth 20 points. Sketch a graph to show his score if he misses 1, 2, 3, 4, or 5 questions.



3-2 Relations and Functions

Give the domain and range of each relation. Tell whether the relation is a function. Explain.

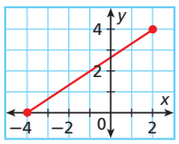
4.



5.

x	−2	−2	0	2	2
y	3	3	3	3	3

6.



3-3 Writing Functions

Determine a relationship between the x- and y-values. Write an equation.

7.

x	1	2	3	4
y	−6	−5	−4	−3

y is 7 less than x; $y = x - 7$.

8.

x	1	2	3	4
y	−3	−6	−9	−12

y is −3 times the value of x; $y = -3x$.

9. A printer can print 8 pages per minute. Identify the dependent and independent variables for the situation. Write an equation in function notation.

ind.: minutes; dep.: pages; $f(m) = 8m$

Evaluate each function for the given input values.

10. For $f(x) = 3x - 1$, find $f(x)$ when $x = 2$. **5**

11. For $g(x) = x^2 - x$, find $g(x)$ when $x = -2$. **6**

12. A photographer charges a sitting fee of $15 plus $3 for each pose. Write a function to describe the situation. Find a reasonable domain and range for up to 5 poses.

$f(x) = 15 + 3x$, D: {1, 2, 3, 4, 5}; R: {18, 21, 24, 27, 30}

3-4 Graphing Functions

Graph each function for the given domain.

13. $2x - y = 3$; D: $\{-2, 0, 1, 3\}$

14. $y = 4 - x^2$; D: $\{-1, 0, 1, 2\}$

15. $y = 3 - 2x$; D: $\{-1, 0, 1, 3\}$

Graph each function.

16. $x + y = 6$

17. $y = |x| - 3$

18. $y = x^2 + 1$

19. The function $y = 8x$ represents how many miles y a certain storm travels in x hours. Graph the function and estimate the number of miles the storm travels in 10.5 h.

READY TO GO ON? SECTION 3A

Organizer

Objective: Assess students' mastery of concepts and skills in this section.

Resources

Assessment Resources
Section Quiz

INTERVENTION

Resources

***Ready to Go On? Intervention and Enrichment* Worksheets**

***Ready to Go On?* Online**

Answers

3. See Additional Answers.
4. D: {−1, 0, 1}; R: {2, 3, 4}; no; the x-value 0 is assigned to the y-value 3 and the y-value 4.
5, 6, 13–19. See Additional Answers.

READY TO GO ON?

Diagnose and Prescribe

NO — INTERVENE

YES — ENRICH

Ready to Go On? Intervention	*Ready to Go On? Intervention*, Section 3A: Worksheets	*Ready to Go On? Intervention*, Section 3A: Online	
Lesson 3-1	3-1 Intervention	Activity 3-1	Diagnose and Prescribe Online
Lesson 3-2	3-2 Intervention	Activity 3-2	
Lesson 3-3	3-3 Intervention	Activity 3-3	
Lesson 3-4	3-4 Intervention	Activity 3-4	

***Ready to Go On? Enrichment*, Section 3A**
Worksheets
Online

SECTION
3B Applying Functions

One-Minute Section Planner

Lesson	Lab Resources	Materials
Lesson 3-5 Scatter Plots and Trend Lines • Create and interpret scatter plots. • Use trend lines to make predictions. ☑ SAT-10 ☑ NAEP ☐ ACT ☑ SAT ☑ SAT Subject Tests	***Algebra Lab Activities*** Algebra Lab (A) Algebra Lab (B) Algebra Lab (C)	**Optional** uncooked spaghetti, centimeter cubes (MK)
Technology Lab Interpret Scatter Plots and Trend Lines • Use a graphing calculator to graph a trend line on a scatter plot. ☑ SAT-10 ☑ NAEP ☐ ACT ☑ SAT ☑ SAT Subject Tests	***Technology Lab Activities*** Lab Recording Sheet	**Required** graphing calculator
Lesson 3-6 Arithmetic Sequences • Recognize and extend an arithmetic sequence. • Find a given term of an arithmetic sequence. ☑ SAT-10 ☑ NAEP ☑ ACT ☑ SAT ☑ SAT Subject Tests		

Note: If NAEP is checked, the content is tested on either the Grade 8 or the Grade 12 NAEP assessment.

MK = *Manipulatives Kit*

Selected Answers in Student Edition

The answers to the following exercises are included in the Selected Answers section at the back of the Student Edition textbook.

Lesson 5: 3, 5, 7, 9, 11, 15, 17, 19, 23, 25, 27a, 27b, 29

Lesson 6: 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33b, 33c, 33d, 35, 37, 39a, 39b, 39c, 39d, 41a, 41b, 41c, 41d, 43, 45, 47a, 47b

All answers for the Check it Out! exercises in each lesson and for the Chapter Study Guide: Review are included in the Student Edition.

Math Background

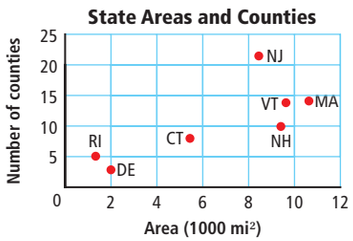
COMMON CORE PROFESSIONAL DEVELOPMENT

USING RELATIONS AND FUNCTIONS

Lesson 3-5

Scatter plots provide an efficient way to present, analyze, and describe large quantities of data. As such, they are one of the most important applications of relations.

A scatter plot is a graph that shows *bivariate data*; that is, data for which there are two variables for each observation. Each point on the scatter plot represents one data pair. The scatter plot below shows the area and the number of counties for the seven smallest U.S. states. Each point represents the data pair for a single state. Rhode Island has an area of approximately 1500 square miles and 5 counties. This is represented by the ordered pair (1.5, 5).



Students are often tempted to connect points on a scatter plot. It is essential that students understand that a scatter plot shows all of the collected data and that a path that is drawn by connecting the points is meaningless.

A *correlation* describes the relationship between the two data sets in a scatter plot. A *positive correlation* means that both sets of data values tend to increase together. The scatter plot of state areas and counties shows a positive correlation. A *negative correlation* means that one set of data values tends to increase as the other decreases. A scatter plot that shows the number of hours a group of people spend reading and the number of hours they spend watching television would likely show a negative correlation.

More specifically, the *correlation coefficient* measures the degree of similarity between two data sets. The correlation coefficient is a value between -1 and 1, where -1 indicates a perfect negative correlation (the data points all fall on a downward slanting line) and 1 indicates a perfect positive correlation (the data points all fall on an upward-slanting line). The closer the coefficient is to -1 or 1, the stronger the correlation, and the more closely the data points resemble a straight line. A coefficient of 0 means the data show *no correlation*, and in this case the scatter plot's points may appear to be randomly distributed.

ARITHMETIC SEQUENCES

Lesson 3-6

A *sequence* is a type of function whose domain is the natural numbers, {1, 2, 3, ...}. Each natural number is paired with a term of the sequence; hence the range is the terms of the sequence. The figure shows how the natural numbers are paired with the terms of the sequence formed by positive multiples of 3.

1	2	3	4	...	n
3	6	9	12	...	
a_1	a_2	a_3	a_4	...	a_n

In an *arithmetic sequence,* all pairs of successive terms differ by the same nonzero constant, called the *common difference.* Equivalently, each term is equal to the previous term plus the common difference. The above sequence is arithmetic with common difference 3.

The nth term of an arithmetic sequence with common difference d is given by $a_n = a_1 + (n - 1)d$. Students are sometimes puzzled by the use of $n - 1$. Exploring simple cases can help students see why $n - 1$ makes sense. For example, to find the 2nd term of the sequence, start with the first term and add the common difference once. To find the 3rd term, start with the first term and add the common difference twice. Generally, to find the nth term, start with the first term and add the common difference $n - 1$ times, as stated in the formula.

3-5 Organizer

Pacing: Traditional 1 day
Block $\frac{1}{2}$ day

Objectives: Create and interpret scatter plots.

Use trend lines to make predictions.

Algebra Labs
In *Algebra Lab Activities*

Online Edition
Tutorial Videos, Interactivity, TechKeys

Countdown

Power Presentations with PowerPoint®

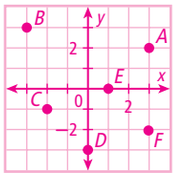
Warm Up

Graph each point.

1. $A(3, 2)$ **2.** $B(-3, 3)$

3. $C(-2, -1)$ **4.** $D(0, -3)$

5. $E(1, 0)$ **6.** $F(3, -2)$



Also available online

Math Humor

Q: What should you title a graph showing the relative diameters and weights of a batch of pancakes?

A: The Batter Plot!

COMMON CORE

CC.9-12.S.ID.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.

CC.9-12.N.Q.1 ... choose and interpret the scale and the origin in graphs and data displays.*

Learn It Online
State Resources Online

COMMON CORE

3-5 Scatter Plots and Trend Lines

CC.9-12.S.ID.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. *Also* **CC.9-12.NQ.1***

Objectives
Create and interpret scatter plots.

Use trend lines to make predictions.

Vocabulary
scatter plot
correlation
positive correlation
negative correlation
no correlation
trend line

Who uses this?
Ecologists can use scatter plots to help them analyze data about endangered species, such as ocelots. (See Example 1.)

In this chapter, you have examined relationships between sets of ordered pairs, or data. Displaying data visually can help you see relationships.

A **scatter plot** is a graph with points plotted to show a possible relationship between two sets of data. A scatter plot is an effective way to display some types of data.

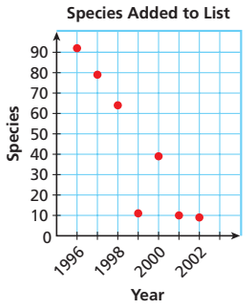
EXAMPLE 1 Graphing a Scatter Plot from Given Data

The table shows the number of species added to the list of endangered and threatened species in the United States during the given years. Graph a scatter plot using the given data.

Increase in List							
Calendar Year	1996	1997	1998	1999	2000	2001	2002
Species	91	79	62	11	39	10	9

Source: U.S. Fish and Wildlife Service

Helpful Hint
The point (2000, 39) tells you that in the year 2000, the list increased by 39 species.



Use the table to make ordered pairs for the scatter plot.

The x-value represents the calendar year and the y-value represents the number of species added.

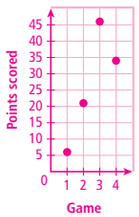
Plot the ordered pairs.

CHECK IT OUT!

1. The table shows the number of points scored by a high school football team in the first four games of a season. Graph a scatter plot using the given data.

Game	1	2	3	4
Score	6	21	46	34

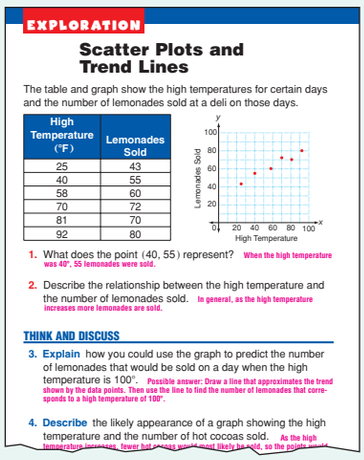
1. Football Team Scores



A **correlation** describes a relationship between two data sets. A graph may show the correlation between data. The correlation can help you analyze trends and make predictions. There are three types of correlations between data.

Wardene Weisser/BRUCE COLEMAN INC./Alamy

1 Introduce


EXPLORATION

Scatter Plots and Trend Lines

The table and graph show the high temperatures for certain days and the number of lemonades sold at a deli on those days.

High Temperature (°F)	Lemonades Sold
25	43
40	55
58	60
70	72
81	70
92	80

1. What does the point (40, 55) represent? When the high temperature was 40°, 55 lemonades were sold.

2. Describe the relationship between the high temperature and the number of lemonades sold. In general, as the high temperature increases more lemonades are sold.

THINK AND DISCUSS

3. **Explain** how you could use the graph to predict the number of lemonades that would be sold on a day when the high temperature is 100°. Possible answer: Draw a line that approximates the trend shown by the data points. Then use the line to find the number of lemonades that corresponds to a high temperature of 100°.

4. **Describe** the likely appearance of a graph showing the high temperature and the number of hot cocoas sold. As the high temperature increases, fewer hot cocoas would most likely be sold, so the points would

Motivate

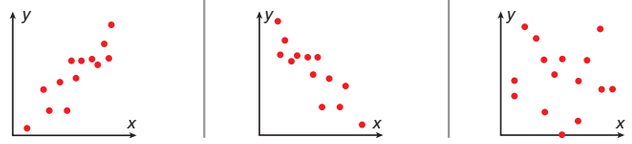
Tell students that when 100 fish were placed in a pond, a fisherman caught 5. The next year, 150 fish were placed in the pond, and the fisherman caught 8. Ask students how many fish they think the fisherman would catch in a year when only 50 fish were placed in the pond. Have students explain their reasoning. Possible answer: 2; the fisherman caught more when there were more fish in the pond, so he is likely to catch fewer when fewer fish are in the pond.

Explorations and answers are provided in your online resources.

Know it! Note

Correlations

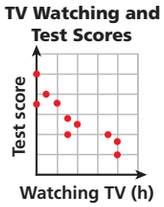
Positive Correlation	Negative Correlation	No Correlation
Both sets of data values increase.	One set of data values increases as the other set decreases.	There is no relationship between the data sets.



In the endangered species graph, as time increases, the number of new species added decreases. So the correlation between the data is negative.

EXAMPLE 2 Describing Correlations from Scatter Plots

Describe the correlation illustrated by the scatter plot.



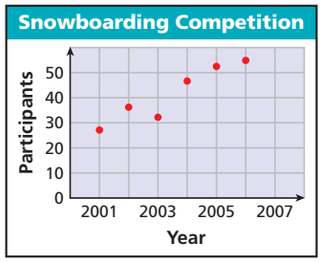
As the number of hours spent watching TV increased, test scores decreased.

There is a negative correlation between the two data sets.

CHECK IT OUT!

2. Describe the correlation illustrated by the scatter plot.

positive



EXAMPLE 3 Identifying Correlations

Identify the correlation you would expect to see between each pair of data sets. Explain.

A **the number of empty seats in a classroom and the number of students seated in the class**

You would expect to see a negative correlation. As the number of students increases, the number of empty seats decreases.

B **the number of pets a person owns and the number of books that person read last year**

You would expect to see no correlation. The number of pets a person owns has nothing to do with how many books the person has read.

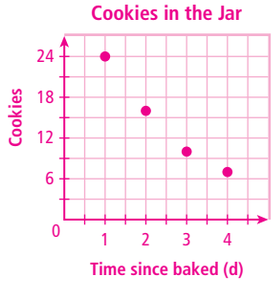
Power Presentations with PowerPoint®

Additional Examples

Example 1

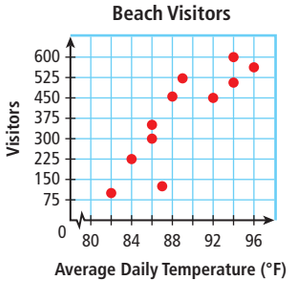
The table shows the number of cookies in a jar from the time since they were baked. Graph a scatter plot using the given data.

Cookies in the Jar				
Time Since Baked (d)	1	2	3	4
Cookies	24	16	10	7



Example 2

Describe the correlation illustrated by the scatter plot.



positive

Example 3

Identify the correlation you would expect to see between each pair of data sets. Explain.

A. The average temperature in a city and the number of speeding tickets given in the city
No correlation; the number of speeding tickets has nothing to do with the temperature.

B. The number of people in an audience and ticket sales
Positive; as ticket sales increase, the number of people in the audience increases.

C. The time since the start of a race and a runner's distance to the finish line
Negative; as the time increases, the distance to the finish line decreases.

Also available online

2 Teach

Guided Instruction

When working through **Example 2,** show students a graph of a perfect positive correlation (points form a straight line), strong positive correlation (points almost form a straight line), and weak positive correlation (generally going up, but points spread out). Then do the same for negative correlation.

Teaching Tip

Visual For smaller graphs, a piece of uncooked spaghetti works well when trying to place a trend line.

Reaching All Learners

Through Kinesthetic Experience

Build a tower of centimeter cubes by adding one cube at random time intervals. Centimeter cubes can be found in the Manipulatives Kit (MK). Have a volunteer record how many cubes are in place every second. Have another student graph a scatter plot of the data. Point out the connection between the growing tower of cubes, the passage of time, and the positive correlation in the scatter plot.

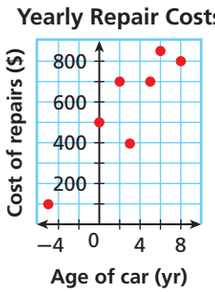
Power Presentations with PowerPoint®

Additional Examples

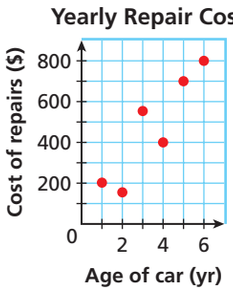
Example 4

Choose the scatter plot that best represents the relationship between the age of a car and the amount of money spent each year on repairs. Explain.

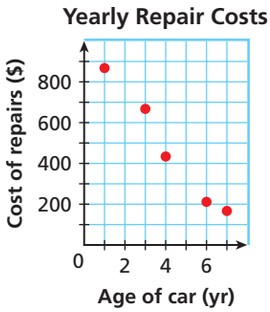
Graph A



Graph B



Graph C



B; graph A shows negative age; graph C shows costs decreasing, a negative correlation.

Also available online

Identify the correlation you would expect to see between each pair of data sets. Explain.

C **the monthly rainfall and the depth of water in a reservoir**

You would expect to see a positive correlation. As more rain falls, there is more water in the reservoir.

CHECK IT OUT!

Identify the correlation you would expect to see between each pair of data sets. Explain.

3a. the temperature in Houston and the number of cars sold in Boston

3b. the number of members in a family and the size of the family's grocery bill

3c. the number of times you sharpen your pencil and the length of your pencil

3a. No correlation; the temperature in Houston has nothing to do with the number of cars sold in Boston.

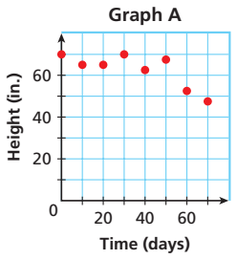
b. Positive; as the number of family members increases, more food is needed, so the grocery bill increases too.

Negative; as the number of times you sharpen your pencil increases, the length of the pencil decreases.

EXAMPLE 4 Matching Scatter Plots to Situations

Choose the scatter plot that best represents the relationship between the number of days since a sunflower seed was planted and the height of the plant. Explain.

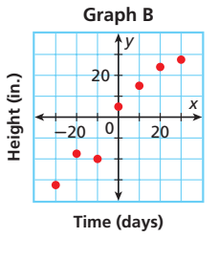
Graph A



There will be a positive correlation between the number of days and the height because the plant will grow each day.

Graph A has a negative correlation, so it is incorrect.

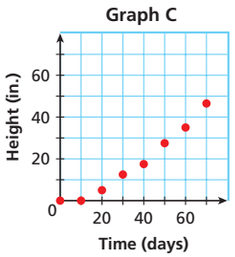
Graph B



Neither the number of days nor the plant heights can be negative.

Graph B shows negative values, so it is incorrect.

Graph C



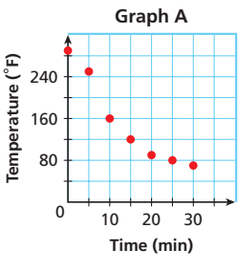
This graph shows all positive coordinates and a positive correlation, so it could represent the data sets.

Graph C is the correct scatter plot.

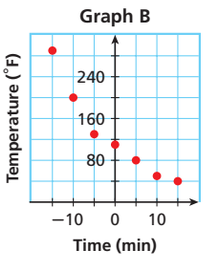
CHECK IT OUT!

4. Choose the scatter plot that best represents the relationship between the number of minutes since a pie has been taken out of the oven and the temperature of the oven. Explain.

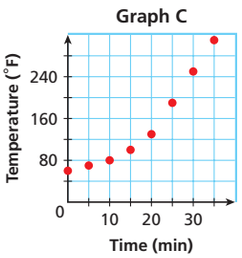
Graph A



Graph B



Graph C



4. Graph A; it cannot be graph B because graph B shows negative minutes; it cannot be graph C because graph C shows the temperature of the oven increasing, a positive correlation.

MATHEMATICAL PRACTICES Questioning Strategies

EXAMPLE 1

- What do the points being graphed represent?

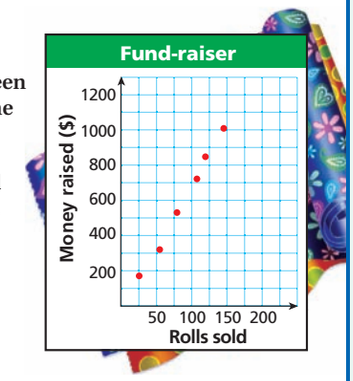
EXAMPLES 2–4

- What does a scatter plot look like if there is no correlation between the data sets? positive correlation? negative correlation?

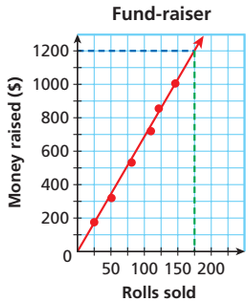
You can graph a line on a scatter plot to help show a relationship in the data. This line, called a **trend line,** helps show the correlation between data sets more clearly. It can also be helpful when making predictions based on the data.

EXAMPLE 5 *Fund-raising Application*

The scatter plot shows a relationship between the total amount of money collected and the total number of rolls of wrapping paper sold as a school fund-raiser. Based on this relationship, predict how much money will be collected when 175 rolls have been sold.



Draw a trend line and use it to make a prediction.



Draw a line that has about the same number of points above and below it. Your line may or may not go through data points.

Find the point on the line whose x-value is 175. The corresponding y-value is 1200.

Based on the data, $1200 is a reasonable prediction of how much money will be collected when 175 rolls have been sold.

CHECK IT OUT! **5.** Based on the trend line above, predict how many wrapping paper rolls need to be sold to raise $500. **about 75 rolls**

MATHEMATICAL PRACTICES

THINK AND DISCUSS

1. Is it possible to make a prediction based on a scatter plot with no correlation? Explain your answer.

Know it! Note

2. GET ORGANIZED Copy and complete the graphic organizer with either a scatter plot, a real-world example, or both.

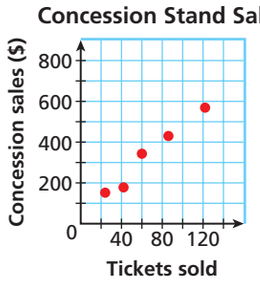
	Graph	Example
Positive Correlation		
Negative Correlation		The amount of water in a watering can and the number of flowers watered
No Correlation		

Power Presentations with PowerPoint®

Additional Examples

Example 5

The scatter plot shows a relationship between the total amount of money collected at the concession stand and the total number of tickets sold at a movie theater. Based on this relationship, predict how much money will be collected at the concession stand when 150 tickets have been sold. about $720



Also available online

MATHEMATICAL PRACTICES **Questioning Strategies**

EXAMPLE 5

- How do you decide where to draw the trend line?

Teaching Tip **Reading Math** Explain to students that in math, *trend* means "tendency," not "fashion." A *trend* line shows where the data *tend* to lie.

3 Close

Summarize

Tell students that in a positive correlation, the values of the variables tend to go in the same direction; that is, as the values of the first variable increase, so do the values of the second variable. In a negative correlation, the opposite is true. As the values of the first variable increase, the values of the second variable tend to decrease. Have students give real-world examples of each type.

ONGOING ASSESSMENT and INTERVENTION

***Diagnose Before* the Lesson**
Warm Up, TE

***Monitor During* the Lesson**
Check It Out! Exercises, SE
Questioning Strategies, TE

***Assess After* the Lesson**
Lesson Quiz, TE
Alternative Assessment, TE

Answers to *Think and Discuss*

1. No; no correlation means that there is no relationship and the points on the graph show no pattern.
2. See Additional Answers.

Assignment Guide

Assign *Guided Practice* exercises as necessary.

If you finished Examples **1–3**
- **Basic** 14–18
- **Average** 14–18
- **Advanced** 14–18

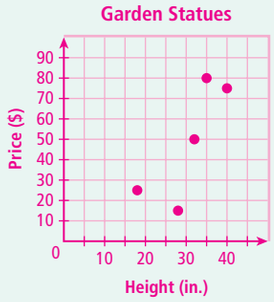
If you finished Examples **1–5**
- **Basic** 14–23, 25–27, 29–31
- **Average** 14–32
- **Advanced** 14–24, 26–33

Homework Quick Check
Quickly check key concepts.
Exercises: 14, 16, 18, 20, 21, 22

Answers

2. Possible answer: A graph with a negative correlation shows one set of values decreasing as the other set increases, while a graph with no correlation shows no relationship between the sets of values.

4.



MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them. Exercises 4, 13–14, 21, 24, 27, 32–33

Reason abstractly and quantitatively. Exercises 5–12, 15–20, 22, 25–26, 28

Construct viable arguments and critique the reasoning of others. Exercise 23

Learn It Online
State Resources Online

3-5 Exercises

Learn It Online
Homework Help Online
Parent Resources Online

GUIDED PRACTICE

3. No; a trend line just fits the pattern of data points, so it usually does not pass through every point.

Vocabulary Apply the vocabulary from this lesson to answer each question.

1. Give an example of a graph that is not a *scatter plot.* Possible answer: a circle graph
2. How is a scatter plot that shows *no correlation* different from a scatter plot that shows a *negative correlation*?
3. Does a *trend line* always pass through every point on a scatter plot? Explain.

SEE EXAMPLE 1

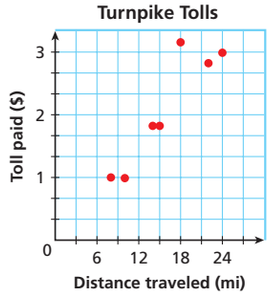
4. Graph a scatter plot using the given data.

Garden Statue	Cupid	Gnome	Lion	Flamingo	Wishing well
Height (in.)	32	18	35	28	40
Price ($)	50	25	80	15	75

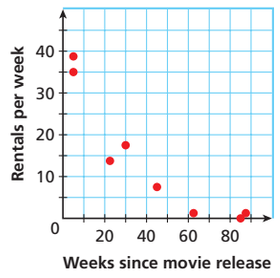
SEE EXAMPLE 2

Describe the correlation illustrated by each scatter plot.

5. negative



6. positive



SEE EXAMPLE 3

Identify the correlation you would expect to see between each pair of data sets. Explain.

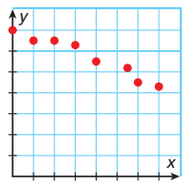
7. the volume of water poured into a container and the amount of empty space left in the container Neg.; as more water is poured, less space remains.
8. a person's shoe size and the length of the person's hair No correlation; there is no relationship.
9. the outside temperature and the number of people at the beach Pos.; as temperature increases, the number of people at the beach increases.

SEE EXAMPLE 4

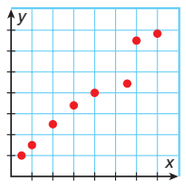
Choose the scatter plot that best represents the described relationship. Explain.

10. age of car and number of miles traveled B; there should be a pos. correlation.
11. age of car and sales price of car A; there should be a neg. correlation.
12. age of car and number of states traveled to C; there should be no correlation.

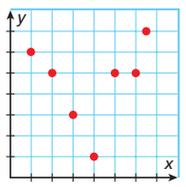
Graph A



Graph B



Graph C



Teacher to Teacher

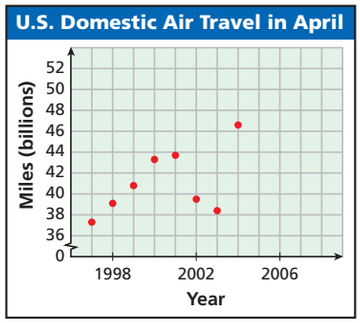
To reinforce the concepts of scatter plots and trend lines, I use an exercise where the students guess the ages of various celebrities. Then students make a scatter plot showing the relationship between their guesses and the actual ages of the celebrities. This helps them to discover the trend line. Because of the topic, students are very involved.

Lendy Jones
Killeen, TX

SEE EXAMPLE 5

13. Transportation The scatter plot shows the total number of miles passengers flew on U.S. domestic flights in the month of April for the years 1997–2004. Based on this relationship, predict how many miles passengers flew in April 2008.

Possible answer: 53 million



PRACTICE AND PROBLEM SOLVING

Independent Practice	
For Exercises	See Example
14	1
15–16	2
17–18	3
19–20	4
21	5

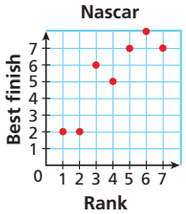
Extra Practice
See Extra Practice for more Skills Practice and Applications Practice exercises.

14. Graph a scatter plot using the given data.

Train Arrival Time	6:45 A.M.	7:30 A.M.	8:15 A.M.	9:45 A.M.	10:30 A.M.
Passengers	160	148	194	152	64

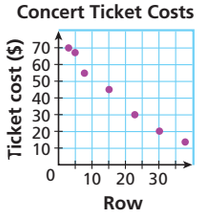
Describe the correlation illustrated by each scatter plot.

15.



positive

16.



negative

17. Pos.; greater speed results in more distance covered in a given time.

Identify the correlation you would expect to see between each pair of data sets. Explain.

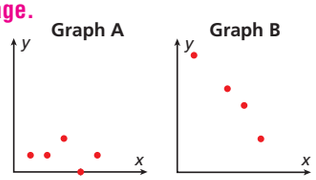
17. the speed of a runner and the distance she can cover in 10 minutes

18. the year a car was made and the total mileage **Neg.; cars made in higher-numbered years are newer and should have lower mileage.**

Choose the scatter plot that best represents the described relationship. Explain.

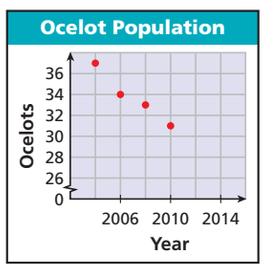
19. the number of college classes taken and the number of roommates **A; there should be no correlation.**

20. the number of college classes taken and the hours of free time. **B; there should be a neg. correlation.**



21. Ecology The scatter plot shows a projection of the average ocelot population living in Laguna Atascosa National Wildlife Refuge near Brownsville, Texas. Based on this relationship, predict the number of ocelots living at the wildlife refuge in 2014 if nothing is done to help manage the ocelot population.

Possible answer: 25



LINK

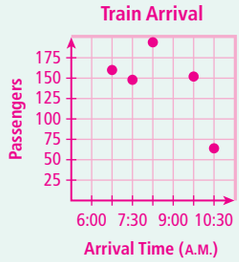
Ecology

The ocelot population in Texas is dwindling due in part to their habitat being destroyed. The ocelot population at Laguna Atascosa National Wildlife Refuge is monitored by following 5–10 ocelots yearly by radio telemetry.

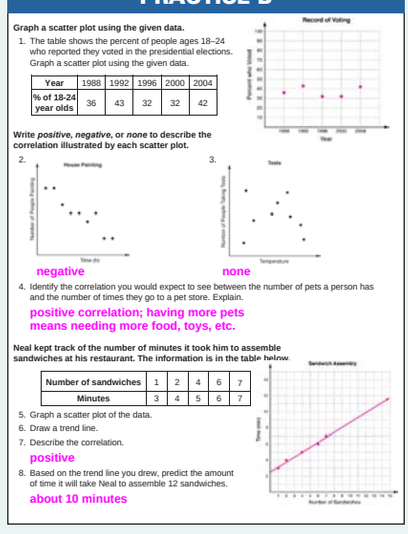
Roy Toft

Answers

14.



PRACTICE A
PRACTICE C
PRACTICE B



Graph a scatter plot using the given data.

1. The table shows the percent of people ages 18–24 who reported they voted in the presidential elections. Graph a scatter plot using the given data.

Year	1988	1992	1996	2000	2004
% of 18-24 year olds	36	43	32	32	42

Write *positive*, *negative*, or *none* to describe the correlation illustrated by each scatter plot.

2. negative

3. none

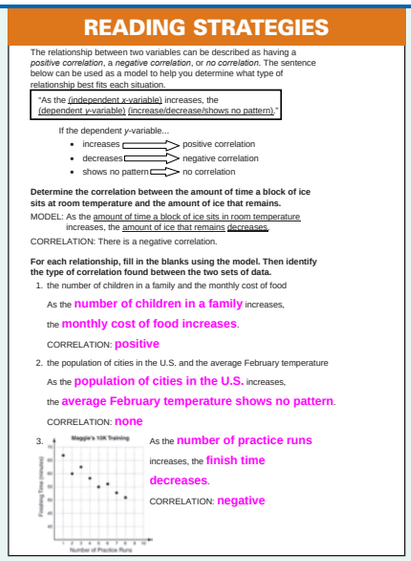
4. Identify the correlation you would expect to see between the number of pets a person has and the number of times they go to a pet store. Explain.
positive correlation; having more pets means needing more food, toys, etc.

Neal kept track of the number of minutes it took him to assemble sandwiches at his restaurant. The information is in the table below.

Number of sandwiches	1	2	4	6	7
Minutes	3	4	5	6	7

5. Graph a scatter plot of the data.
6. Draw a trend line.
7. Describe the correlation.
positive
8. Based on the trend line you drew, predict the amount of time it will take Neal to assemble 12 sandwiches.
about 10 minutes

READING STRATEGIES



The relationship between two variables can be described as having a *positive correlation*, a *negative correlation*, or *no correlation*. The sentence below can be used as a model to help you determine what type of relationship best fits each situation.

"As the (independent *x*-variable) increases, the (dependent *y*-variable) (increase/decrease/shows no pattern)."

If the dependent *y*-variable...
- increases ⇒ positive correlation
- decreases ⇒ negative correlation
- shows no pattern ⇒ no correlation

Determine the correlation between the amount of time a block of ice sits at room temperature and the amount of ice that remains.

MODEL: As the amount of time a block of ice sits in room temperature increases, the amount of ice that remains decreases.

CORRELATION: There is a negative correlation.

For each relationship, fill in the blanks using the model. Then identify the type of correlation found between the two sets of data.

1. the number of children in a family and the monthly cost of food

As the **number of children in a family** increases, the **monthly cost of food increases**.

CORRELATION: **positive**

2. the population of cities in the U.S. and the average February temperature

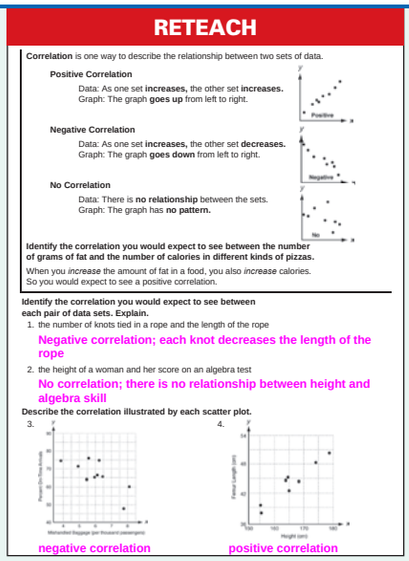
As the **population of cities in the U.S.** increases, the **average February temperature shows no pattern**.

CORRELATION: **none**

3. As the **number of practice runs** increases, the **finish time decreases**.

CORRELATION: **negative**

RETEACH



Correlation is one way to describe the relationship between two sets of data.

Positive Correlation
Data: As one set **increases,** the other set **increases.**
Graph: The graph **goes up** from left to right.

Negative Correlation
Data: As one set **increases,** the other set **decreases.**
Graph: The graph **goes down** from left to right.

No Correlation
Data: There is **no relationship** between the sets.
Graph: The graph has **no pattern.**

Identify the correlation you would expect to see between the number of grams of fat and the number of calories in different kinds of pizzas.

When you *increase* the amount of fat in a food, you also *increase* calories.
So you would expect to see a positive correlation.

Identify the correlation you would expect to see between each pair of data sets. Explain.

1. the number of knots tied in a rope and the length of the rope
Negative correlation; each knot decreases the length of the rope

2. the height of a woman and her score on an algebra test
No correlation; there is no relationship between height and algebra skill

Describe the correlation illustrated by each scatter plot.

3. **negative correlation**

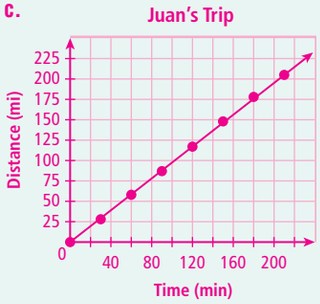
4. **positive correlation**

MULTI-STEP TEST PREP **Exercise 27** involves using a trend line to make a prediction. This exercise prepares students for the Multi-Step Test Prep.

Answers

26. Fewer people will buy an item if the price goes up. So there is a negative correlation between the price of an item and the number of people buying the item.

27a, c.

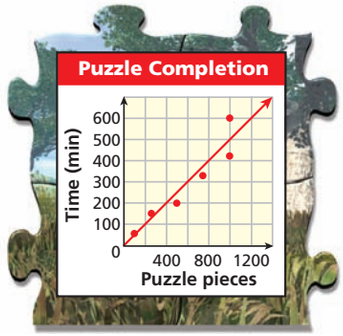


b. Positive; as time increases, the number of miles also increases.

c. Possible answer: about 235 mi

23. Pos.; as the number of left shoes sold increases, the number of right shoes sold also increases, because people need shoes for both feet.

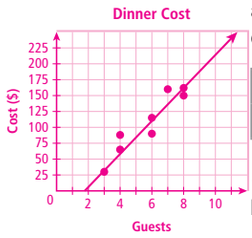
22. Estimation Angie enjoys putting jigsaw puzzles together. The scatter plot shows the number of puzzle pieces and the time in minutes it took her to complete each of her last six puzzles. Use the trend line to estimate the time in minutes it will take Angie to complete a 1200-piece puzzle.



Possible answer: 600 min

23. Critical Thinking Describe the correlation between the number of left shoes sold and the number of right shoes sold.

24a, c.



24. Roma had guests for dinner at her house eight times and has recorded the number of guests and the total cost for each meal in the table.

Guests	3	4	4	6	6	7	8	8
Cost ($)	30	65	88	90	115	160	150	162

a. Graph a scatter plot of the data.

b. Describe the correlation.

c. Draw a trend line.

d. Based on the trend line you drew, predict the cost of dinner for 11 guests.

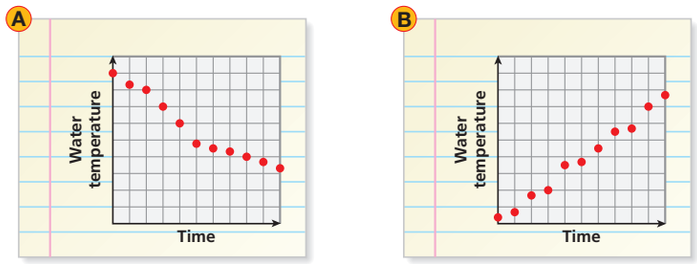
e. What if...? Suppose that each cost in the table increased by $5. How will this affect the cost of dinner for 11 guests?

b. Pos.; as the number of guests increases, the cost increases.

d. Possible answer: about $230

e. The cost will increase.

25. ERROR ANALYSIS Students graphed a scatter plot for the temperature of hot bath water and time if no new water is added. Which graph is incorrect? Explain the error.



25. B; the bath water is probably hot at first, but as time passes, the temperature decreases.

26. Critical Thinking Will more people or fewer people buy an item if the price goes up? Explain the relationship and describe the correlation.

MULTI-STEP TEST PREP

27. Juan and his parents are visiting a university 205 miles from their home. As they travel, Juan uses the car odometer and his watch to keep track of the distance.

a. Make a scatter plot for this data set.

b. Describe the correlation. Explain.

c. Draw a trend line for the data and predict the distance Juan would have traveled going to a university 4 hours away.

Time (min)	Distance (mi)
0	0
30	28
60	58
90	87
120	117
150	148
180	178
210	205

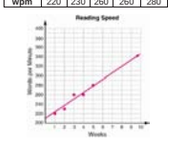
(bl),comstock/Getty Photos; (tr),©COMSTOCK, Inc.

PROBLEM SOLVING

Fawn is trying to improve her reading skills by taking a speed-reading class. She is measuring how many words per minute (wpm) she can read after each week of the class.

1. Graph a scatter plot using the given data.

Weeks	1	2	3	4	5
wpm	220	230	260	260	280



2. Describe the correlation illustrated by the scatter plot.
positive correlation

3. Draw a trend line and use it to predict the number of words per minute that Fawn will read after 8 weeks of this class.
about 320 wpm

4. Fawn is paying for this class each week out of her savings account. Identify the correlation between the number of classes and Fawn's account balance.
negative correlation

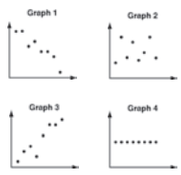
Choose the scatter plot that best represents the described relationship.

5. the distance a person runs and how physically tired that person is
A Graph 1 C Graph 3
B Graph 2 D Graph 4

6. the price of a new car and the number of hours in a day
F Graph 1 H Graph 3
G Graph 2 J Graph 4

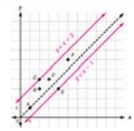
7. a person's age and the amount of broccoli the person eats
A Graph 1 C Graph 3
B Graph 2 D Graph 4

8. the number of cats in a barn and the number of mice in that barn
F Graph 1 H Graph 3
G Graph 2 J Graph 4



CHALLENGE

A scatter plot that represents a data set has many advantages. When you visualize the data on the coordinate plane, you can draw an inference about whether there may be a correlation, positive or negative. You may also be able to draw the inference that no correlation is apparent.
In the graph at right, you can see a scatter plot for the data set below.



(1, 1), (2, 3), (2, 4), (3, 4), (4, 3), and (5, 6)

The scatter plot shows a positive correlation between x and y. The points cluster closely to the line $y = x$. The greatest difference in the y-coordinates of the data points and the corresponding y-coordinates of the graph of $y = x$ is 2.

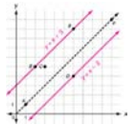
Refer to the graph above.

1. a. On the coordinate grid above, graph $y = x + 2$.
b. On the coordinate grid above, graph $y = x - 1$.
c. Do the graphs that you sketched in parts **a** and **b** bound or enclose all the given data points? Explain your response.
Yes, all the data points are either on the lines or between them.

The graph at right shows a scatter plot and the line $y = x$.

In Exercises 2 and 3, refer to the graph at right.

2. a. Find the greatest difference in the y-coordinates of the data points above the graph of $y = x$ and the corresponding y-coordinates of the graph of $y = x$. 3 units
b. Find the greatest difference in the y-coordinates of the data points below the graph of $y = x$ and the corresponding y-coordinates of the graph of $y = x$. 2 units



3. Using your answers from Exercise 2, sketch lines that are parallel to $y = x$ and that bound all the data points.

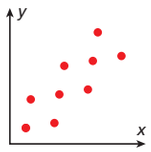
4. Explain how you can use bounding lines to analyze a data set with a positive correlation.
The closer together the parallel bounding lines are, the more tightly the data cluster around a line. The farther apart the parallel bounding lines are, the more loosely the data points cluster around a line.

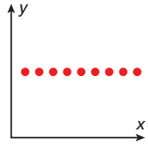
28. Write About It Conduct a survey of your classmates to find the number of siblings they have and the number of pets they have. Predict whether there will be a positive, negative, or no correlation. Then graph the data in a scatter plot. What is the relationship between the two data sets? Was your prediction correct?

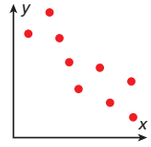
Check students' work.

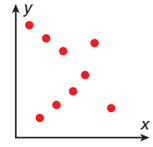
TEST PREP

29. Which graph is the best example of a negative correlation?

Ⓐ 

Ⓑ 

Ⓒ 

Ⓓ 

30. Which situation best describes a positive correlation?

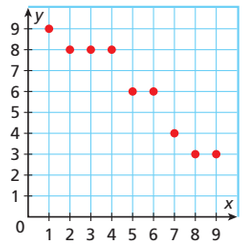
Ⓕ The amount of rainfall on Fridays

Ⓖ The height of a candle and the amount of time it stays lit

Ⓗ The price of a pizza and the number of toppings added

Ⓙ The temperature of a cup of hot chocolate and the length of time it sits

31. Short Response Write a real-world situation for the graph. Explain your answer.



31. Possible answer: The number of milk cartons at the grocery store varies with the number of people who pass by the milk. Each person who passes by either takes milk or does not. If the person does not, the same number of cartons remain.

CHALLENGE AND EXTEND

32. Describe a situation that involves a positive correlation. Gather data on the situation. Make a scatter plot showing the correlation. Use the scatter plot to make a prediction. Repeat for a negative correlation and for no correlation.

Check students' work.

33. Research an endangered or threatened species in your state. Gather information on its population for several years. Make a scatter plot using the data you gather. Is there a positive or negative correlation? Explain. Draw a trend line and make a prediction about the species population over the next 5 years. Check students' work.

TEST PREP DOCTOR Students who choose **B** or **D** for **Exercise 29** may be confusing no correlation with negative correlation.

Students who chose **G** or **J** in **Exercise 30** chose situations that describe negative correlations.

Journal

Have students describe how to identify positive and negative correlations from a scatter plot.

ALTERNATIVE ASSESSMENT

Have students create a scatter plot showing population over time for an endangered species and predict its population in 2020.

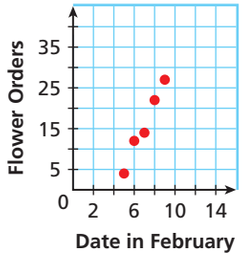
Power Presentations with PowerPoint®

3-5 Lesson Quiz

For Items 1 and 2, identify the correlation you would expect to see between each pair of data sets. Explain.

1. The outside temperature in the summer and the cost of the electric bill Positive; as the outside temperature increases, the electric bill increases because of the use of the air conditioner.
2. The price of a car and the number of passengers it seats No correlation; a very expensive car could seat only 2 passengers.
3. The scatter plot shows the number of orders placed for flowers before Valentine's Day at one shop. Based on this relationship, predict the number of flower orders placed on February 12. about 45

Valentine's Day Orders



Also available online

Technology Organizer

Use with the lesson *Scatter Plots and Trend Lines*

Pacing:
Traditional $\frac{1}{2}$ day
Block $\frac{1}{4}$ day

Objective: Use a graphing calculator to graph a trend line on a scatter plot.

Materials: graphing calculator

Online Edition
Graphing Calculator

Countdown

Resources

Technology Lab Activities
Lab Recording Sheet

Teach

Discuss

When viewing the values of a and b in the equation of the trend line $y = ax + b$, point out that $0.5079441502x - 26.78767453 = 0.5079441502x + (-26.78767453)$.

Close

Key Concept

A trend line can help you interpret a scatter plot.

Assessment

Journal Have students outline the steps for using a calculator to find the equation of a trend line.

Answer to *Try This*

1. For graph, see Additional Answers.

MATHEMATICAL PRACTICES

Use appropriate tools strategically.

COMMON CORE

CC.9-12.S.ID.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.
a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data.
c. Fit a linear function for a scatter plot that suggests a linear association.

Learn It Online
State Resources Online

3-5 Technology LAB

Interpret Scatter Plots and Trend Lines

You can use a graphing calculator to graph a trend line on a scatter plot.

Use with Scatter Plots and Trend Lines

CC.9-12.S.ID.6 Represent data on two quantitative variables on a scatter plot, …. *Also* **CC.9-12.S.ID.6a, CC.9-12.S.ID.6c**

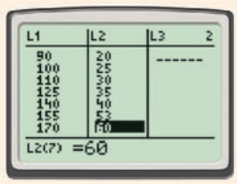
Learn It Online
Lab Resources Online

MATHEMATICAL PRACTICES **Use appropriate tools strategically.**

Activity

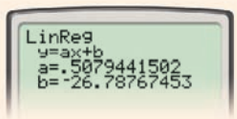
The table shows the recommended dosage of a particular medicine as related to a person's weight. Graph a scatter plot of the given data. Draw the trend line. Then predict the dosage for a person weighing 240 pounds.

Weight (lb)	90	100	110	125	140	155	170	180	200
Dosage (mg)	20	25	30	35	40	53	60	66	75

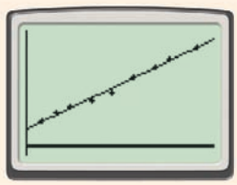


1. First enter the data. Press STAT and select **1: Edit**. In **L1**, enter the first weight. Press ENTER. Continue entering all weights. Use ▶ to move to **L2**. Enter the first dosage. Press ENTER. Continue entering all dosages.

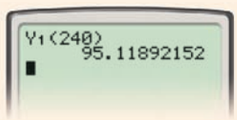
2. To view the scatter plot, press 2nd Y=. Select **Plot 1**. Select **On**, the first plot type, and the plot mark **+**. Press ZOOM. Select **9: ZoomStat**. You should see a scatter plot of the data.



3. To find the trend line, press STAT and select the **CALC** menu. Select **LinReg (ax+b)**. Press ENTER. This gives you the values of a and b in the trend line.



4. To enter the equation for the trend line, press Y=, and then input **.5079441502x − 26.78767453**. Press GRAPH.



5. Now predict the dosage for a weight of 240 pounds. Press VARS. Select **Y-VARS** menu and select **1:Function**. Select **1:Y1**. Enter **(240)**. Press ENTER. The dosage is about 95 milligrams.

Try This

1. The table shows the price of a stock over an 8-month period. Graph a scatter plot of the given data. Draw the trend line. Then predict what the price of one share of stock will be in the twelfth month.

Month	1	2	3	4	5	6	7	8
Price ($)	32	35	37	41	46	50	54	59

about $73.54

Teaching Tip **Technology** Point out to students that **Xlist** gives the name of the list of data used for the x-values of the scatter plot and that **Ylist** gives the name of the list used for the y-values of the scatter plot. Students can change the list names to L1 and L2 by pressing 2nd 1 (L1) and 2nd 2 (L2).

Teaching Tip **Technology** Instead of entering the equation for the trend line manually, students can press Y= VARS, select **5:Statistics**, scroll right to **EQ**, and then select **1:RegEQ**.

Median-Fit Line

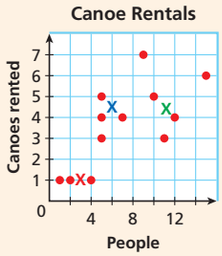
You have learned about trend lines. Now you will learn about another line of fit called the *median-fit line.*

Example

At a water raft rental shop, a group of up to four people can rent a single raft. The table shows the number of rafts rented to different groups of people one morning. Graph the median-fit line for the data.

People x	1	2	4	5	5	5	7	9	10	11	12	15
Rafts Rented y	1	1	1	3	4	5	4	7	5	3	4	6

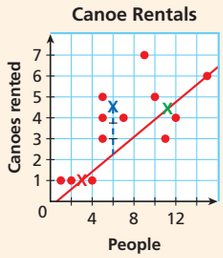
1. Plot the points on a coordinate plane.



2. Divide the data into three sections of equal size. Find the medians of the x-values and the y-values for each section. Plot the three median points with an X.

1	2	4	5	5	5	7	9	10	11	12	15
1	1	1	3	4	5	4	7	5	3	4	6

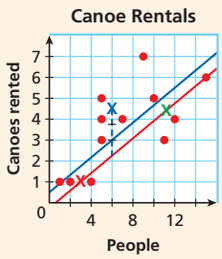
Median point: (3, 1) **Median point: (6, 4.5)** **Median point: (11.5, 4.5)**



3. Connect the outside, or first and third, median points with a line.

4. Lightly draw a dashed line straight down from the middle median point to the line just drawn. Mark the dashed line to create three equal segments.

5. Keeping your ruler parallel to the first line you drew, move your ruler to the mark closest to the line. Draw the line. This is the median-fit line.



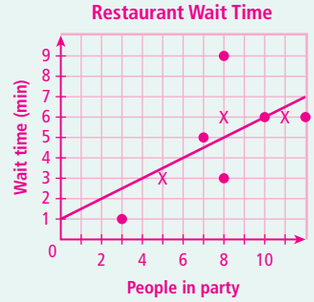
Try This

1. A manager at a restaurant kept track one afternoon of the number of people in a party and the time it took to seat them. Graph the median-fit line for the data.
2. Use your median-fit line to predict the time it would take to seat a party of 6. **4 min**

People x	3	7	8	8	10	12
Wait Time y (min)	1	5	3	9	6	6

Answers to *Try This*

1.



Organizer

Pacing:
Traditional 1 day
Block $\frac{1}{2}$ day

Objective: Apply the algebra skills learned in *Scatter Plots and Trend Lines* to approximate a median-fit line.

Online Edition

Teach

Remember

Students review and reinforce the concept of finding the median, plotting points on a coordinate plane, and drawing trend lines.

Remind students that for an ordered data set with an odd number of values, the median is the middle value. For an ordered data set with an even number of values, the median is the average of the two middle values.

Point out to students that only one set of data values can be ordered from least to greatest. In this case, x-values are ordered, not y-values.

Close

Assess

Using the data in the example, have students demonstrate finding the median points for the second and third sections.

Learn It Online
State Resources Online

3-6 Organizer

Pacing: Traditional 1 day
Block $\frac{1}{2}$ day

Objectives: Recognize and extend an arithmetic sequence.

Find a given term of an arithmetic sequence.

Online Edition
Tutorial Videos

Countdown

Power Presentations with PowerPoint®

Warm Up

Simplify.

1. $5 + (-7)$ -2
2. $\frac{2}{3} + \frac{1}{3} + \frac{1}{3}$ $\frac{4}{3}$
3. $5.3 + 0.8$ 6.1
4. $6(4 - 1)$ 18
5. $-3(2 - 5)$ 9
6. $\frac{12}{5}\left(\frac{10}{3} + \frac{5}{4}\right)$ 11

Evaluate.

7. $\frac{3}{4} + h$ for $h = -2$ $-\frac{5}{4}$
8. $n - 2.8$ for $n = 5.1$ 2.3
9. $6(x - 1)$ for $x = 5$ 24
10. $10 + (5 - 1)s$ for $s = -4$ -6

Also available online

Math Humor

Teacher: Is something wrong with your printer? Your history paper is covered in dots.

Student: Those are ellipses. After the first paragraph, imagine the rest!

COMMON CORE

CC.9-12.F.IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.
CC.9-12.F.BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations….*
CC.9-12.F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input- output pairs (include reading these from a table).*

Learn It Online
State Resources Online

COMMON CORE

3-6 Arithmetic Sequences

CC.9-12.F.IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *Also* **CC.9-12.F.BF.2***, **CC.9-12.F.LE.2***

Objectives
Recognize and extend an arithmetic sequence.

Find a given term of an arithmetic sequence.

Vocabulary
sequence
term
arithmetic sequence
common difference

Why learn this?

The distance between you and a lightning strike can be approximated by using an arithmetic sequence.

During a thunderstorm, you can estimate your distance from a lightning strike by counting the number of seconds from the time you see the lightning until the time you hear the thunder.

Time (s)	Distance (mi)	
1	0.2	
2	0.4	+ 0.2
3	0.6	+ 0.2
4	0.8	+ 0.2
5	1.0	+ 0.2
6	1.2	+ 0.2
7	1.4	+ 0.2
8	1.6	+ 0.2

When you list the times and distances in order, each list forms a *sequence*. A **sequence** is a list of numbers that may form a pattern. Each number in a sequence is a **term**.

In the distance sequence, each distance is 0.2 mi greater than the previous distance. When the terms of a sequence differ by the same nonzero number d, the sequence is an **arithmetic sequence** and d is the **common difference**. The distances in the table form an arithmetic sequence with $d = 0.2$.

The variable a is often used to represent terms in a sequence. The variable a_9, read "a sub 9," is the ninth term in a sequence. To designate any term, or the nth term, in a sequence, you write a_n, where n can be any number.

To find a term in an arithmetic sequence, add d to the previous term.

Know it! Note

Finding a Term of an Arithmetic Sequence

The nth term of an arithmetic sequence with **common difference** d is

$$a_n = a_{n-1} + d.$$

EXAMPLE 1 Identifying Arithmetic Sequences

Determine whether each sequence appears to be an arithmetic sequence. If so, find the common difference and the next three terms in the sequence.

A 12, 8, 4, 0, …

Step 1 Find the difference between successive terms.

12, 8, 4, 0, …
-4 -4 -4

Add −4 to each term to find the next term. The common difference is −4.

Step 2 Use the common difference to find the next 3 terms.

12, 8, 4, 0, -4, -8, -12
-4 -4 -4

$a_n = a_{n-1} + d$

The sequence appears to be an arithmetic sequence with a common difference of −4. The next 3 terms are −4, −8, −12.

Reading Math

The three dots at the end of a sequence are called an ellipsis. They mean that the sequence continues and can be read as "and so on."

1 Introduce

EXPLORATION

Arithmetic Sequences

Mei collects postcards. On August 1, she has 10 cards in her collection. She decides to buy three cards every day thereafter for the rest of the month.

1. The table shows the number of postcards in Mei's collection. Complete the table for the first six days of August.

Day of Month	1	2	3	4	5	6
Postcards	10	13	16	19	22	25

2. How many postcards will Mei have on August 10 (day 10)? 37
3. How many postcards will Mei have on August 15 (day 15)? 52
4. Find a shortcut that you can use to find the number of postcards Mei will have at the end of the month (day 31). 100

THINK AND DISCUSS

5. **Explain** how you found the shortcut to determine the number of postcards Mei will have on day 31. Possible answer: The number of postcards on any given day is 10 plus the product of 3 and the day of the month minus 1.
6. **Describe** a general rule you can use to find the number of postcards Mei will have on day n. Possible answer: $10 + 3(n - 1)$

Motivate

Display the following list of numbers. Ask students to find the next 3 numbers.

1, 2, 4, 8, 16, _, _, _

Tell students that this list of numbers is a *sequence* because it is a list of numbers that form a pattern. Explain that in this lesson they will learn about a type of sequence called an arithmetic sequence.

Explorations and answers are provided in your online resources.

B 1, 4, 9, 16, …

Find the difference between successive terms.

1, 4, 9, 16, …

+3 +5 +7

The difference between successive terms is not the same.

This sequence is not an arithmetic sequence.

CHECK IT OUT! **Determine whether each sequence appears to be an arithmetic sequence. If so, find the common difference and the next three terms.**

1a. $-\frac{3}{4}, -\frac{1}{4}, \frac{1}{4}, \frac{3}{4}, \ldots$ yes; $\frac{1}{2}, \frac{5}{4}, \frac{7}{4}, \frac{9}{4}$

1b. −4, −2, 1, 5, … no

To find the *n*th term of an arithmetic sequence when *n* is a large number, you need an equation or rule. Look for a pattern to find a rule for the sequence below.

1	2	3	4…	*n*	← Position
↓	↓	↓	↓		
3,	5,	7,	9…		← Term
a_1	a_2	a_3	a_4	a_n	

The sequence starts with 3. The common difference *d* is 2. You can use the first term and the common difference to write a rule for finding a_n.

Words	Numbers	Algebra
1st term	3	a_1
2nd term = 1st term plus common difference	3 + (1)2 = 5	$a_1 + 1d$
3nd term = 1st term plus 2 common differences	3 + (2)2 = 7	$a_1 + 2d$
4th term = 1st term plus 3 common differences	3 + (3)2 = 9	$a_1 + 3d$
⋮	⋮	⋮
*n*th term = 1st term plus $(n - 1)$ common differences	$3 + (n - 1)2$	$a_1 + (n - 1)d$

The pattern in the table shows that to find the *n*th term, add the first term to the product of $(n - 1)$ and the common difference.

Know it! Note

Finding the *n*th Term of an Arithmetic Sequence

The *n*th term of an arithmetic sequence with common difference *d* and first term a_1 is

$$a_n = a_1 + (n - 1)d.$$

EXAMPLE 2 Finding the *n*th Term of an Arithmetic Sequence

Find the indicated term of each arithmetic sequence.

A 22nd term: 5, 2, −1, −4, …

Step 1 Find the common difference.

5, 2, −1, −4, …

−3 −3 −3

The common difference is −3.

COMMON ERROR ALERT

Students sometimes get confused when finding the *n*th term. Tell students that if they are finding the 6th term, they will be multiplying the common difference *d* by 5, 6 − 1. The reason is that the formula uses the first term of a sequence as a constant. Ask students what number they would multiply the common difference by if they were looking for the 20th term of an arithmetic sequence. 19

Power Presentations with PowerPoint®

Additional Examples

Example 1

Determine whether each sequence appears to be an arithmetic sequence. If so, find the common difference and the next three terms.

A. 9, 13, 17, 21, . . . yes; 4; 25, 29, 33

B. 10, 8, 5, 1, . . . no

Example 2

Find the indicated term of each arithmetic sequence.

A. 16th term: 4, 8, 12, 16, . . . 64

B. 25th term: $a_1 = -5$; $d = -2$ −53

Also available online

2 Teach

Guided Instruction

Remind students that they had to look for a pattern when they were writing equations earlier in this chapter. Tell them that they will also look for a pattern when identifying an arithmetic sequence. Then review the Distributive Property and the order of operations with students. When teaching the formula for the *n*th term in an arithmetic sequence, point out that students will need to calculate $n - 1$ first because it is in parentheses.

Reaching All Learners Through Cognitive Strategies

ENGLISH LANGUAGE LEARNERS

Challenge students to identify the best way to memorize vocabulary terms and formulas like the one on this page. Suggest the following methods:

- Writing the information over and over again.
- Making up a song or chant.
- Creating flash cards to use with a partner.
- Writing sentences with the vocabulary and working problems with the formulas.

Questioning Strategies

MATHEMATICAL PRACTICES

EXAMPLE 1

- How is identifying an arithmetic sequence similar to identifying a function rule?

EXAMPLE 2

- How do you know what number to substitute for *d*?
- How do you know what number to substitute for *n*?

Teaching Tip **Math Background** A sequence is said to be *recursively defined* if the *n*th term is defined by using the terms that precede it. The box shows the recursive definition of an arithmetic sequence. The box on this page shows the *iterative* definition.

Step 2 Find the 22nd term.

$a_n = a_1 + (n-1)d$ — *Write the rule to find the nth term.*
$a_{22} = 5 + (22-1)(-3)$ — *Substitute 5 for a_1, 22 for n, and –3 for d.*
$= 5 + (21)(-3)$ — *Simplify the expression in parentheses.*
$= 5 - 63$ — *Multiply.*
$= -58$ — *Subtract.*

B 15th term: $a_1 = 7; d = 3$

$a_n = a_1 + (n-1)d$ — *Write the rule to find the nth term.*
$a_{15} = 7 + (15-1)3$ — *Substitute 7 for a_1, 15 for n, and 3 for d.*
$= 7 + (14)3$ — *Simplify the expression in parentheses.*
$= 7 + 42$ — *Multiply.*
$= 49$ — *Add.*

CHECK IT OUT! **Find the indicated term of each arithmetic sequence.**

2a. 60th term: 11, 5, −1, −7, … **−343**

2b. 12th term: $a_1 = 4.2; d = 1.4$ **19.6**

EXAMPLE 3 *Travel Application*

The odometer on a car reads 60,473 on day 1. Every day, the car is driven 54 miles. If this pattern continues, what is the odometer reading on day 20?

Notice that the sequence for the situation is arithmetic with $d = 54$ because the odometer reading will increase by 54 miles per day.

Since the odometer reading on day 1 is 60,473 miles, $a_1 = 60{,}473$.

Since you want to find the odometer reading on day 20, you will need to find the 20th term of the sequence, so $n = 20$.

$a_n = a_1 + (n-1)d$ — *Write the rule to find the nth term.*
$a_{20} = 60{,}473 + (20-1)54$ — *Substitute 60,473 for a_1, 54 for d, and 21 for n.*
$= 60{,}473 + (19)54$ — *Simplify the expression in parentheses.*
$= 60{,}473 + 1026$ — *Multiply.*
$= 61{,}499$ — *Add.*

The odometer will read 61,499 miles on day 20.

CHECK IT OUT! **3.** Each time a truck stops, it drops off 250 pounds of cargo. After stop 1, its cargo weighed 2000 pounds. How much does the load weigh after stop 6? **750 lb**

THINK AND DISCUSS

1. Explain how to determine if a sequence appears to be arithmetic.

2. GET ORGANIZED Copy and complete the graphic organizer with steps for finding the *n*th term of an arithmetic sequence.

Finding the *n*th Term of an Arithmetic Sequence → 1. → 2.

Power Presentations with PowerPoint®

Additional Examples

Example 3

A bag of cat food weighs 18 pounds at the beginning of day 1. Each day, the cats are fed 0.5 pound of food. How much does the bag of cat food weigh at the beginning of day 30? 3.5 lb

Also available online

Questioning Strategies

EXAMPLE 3

- How can you identify *d* in a word problem?
- How do you solve the problem once you know the values of a_1, *d*, and *n*?

Reading Math Point out the key words in **Example 3** that will help students put the information in a formula. Tell students that *each* or *every* with a time word such as *day* or *hour* signals that the following number is *d*. Explain that the number to substitute for *n* may be indicated by an ordinal number. The first term of the sequence is a number that can be associated with the words *at the start* or *first*.

ENGLISH LANGUAGE LEARNERS

3 Close

Summarize

Display the formula $a_n = a_1 + (n-1)d$. Have students define each variable in the formula. Then have them find the value of each variable and the 24th term of the following arithmetic sequence.

10, 4, −2, −8, . . .

In the formula, a_n is the *n*th term, a_1 is the first term, *n* is the term number, and *d* is the common difference. $a_1 = 10$, $n = 24$, $d = -6$, $a_{24} = -128$.

ONGOING ASSESSMENT and INTERVENTION

***Diagnose Before* the Lesson**
Warm Up, TE

***Monitor During* the Lesson**
Check It Out! Exercises, SE
Questioning Strategies, TE

***Assess After* the Lesson**
Lesson Quiz, TE
Alternative Assessment, TE

Answers to *Think and Discuss*

Possible answers:

1. If the sequence has a common difference, it could be an arithmetic sequence.
2. See Additional Answers.

Learn It Online
Homework Help Online
Parent Resources Online

GUIDED PRACTICE

1. **Vocabulary** When trying to find the nth term of an arithmetic sequence you must first know the _____?_____. (*common difference* or *sequence*) **common difference**

SEE EXAMPLE 1

Multi-Step Determine whether each sequence appears to be an arithmetic sequence. If so, find the common difference and the next three terms.

2. 2, 8, 14, 20, … **yes; 6; 26, 32, 38**
3. 2.1, 1.4, 0.7, 0, … **yes; −0.7; −0.7, −1.4, −2.1**
4. 1, 1, 2, 3, … **no**
5. 0.1, 0.3, 0.9, 2.7, … **no**

SEE EXAMPLE 2

Find the indicated term of each arithmetic sequence.

6. 21st term: 3, 8, 13, 18, … **103**
7. 18th term: $a_1 = -2$; $d = -3$ **−53**

SEE EXAMPLE 3

8. **Shipping** To package and ship an item, it costs $5.75 for the first pound and $0.75 for each additional pound. What is the cost of shipping a 12-pound package? **$14**

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
9–12	1
13–14	2
15	3

Extra Practice
See Extra Practice for more Skills Practice and Applications Practice exercises.

Multi-Step Determine whether each sequence appears to be an arithmetic sequence. If so, find the common difference and the next three terms.

9. −1, 10, −100, 1,100, … **no**
10. 0, −2, −4, −6, … **yes; −2; −8, −10, −12**
11. −22, −31, −40, −49, … **yes; −9; −58, −67, −76**
12. 0.2, 0.5, 0.9, 1.1, … **no**

Find the indicated term of each arithmetic sequence.

13. 31st term: 1.40, 1.55, 1.70, … **5.9**
14. 50th term: $a_1 = 2.2$; $d = 1.1$ **56.1**
15. **Travel** Rachel signed up for a frequent-flier program. She receives 4300 frequent-flier miles for her first round trip and 1300 frequent-flier miles for each additional round-trip. How many frequent-flier miles will she have after 5 round-trips? **9500 mi**

Find the common difference for each arithmetic sequence.

16. 0, 6, 12, 18, … **6**
17. $\frac{1}{2}, \frac{3}{4}, 1, \frac{5}{4}, \ldots$ **$\frac{1}{4}$**
18. 107, 105, 103, 101, … **−2**
19. 7.9, 5.7, 3.5, 1.3, … **−2.2**
20. $\frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, \ldots$ **$\frac{1}{5}$**
21. 4.25, 4.32, 4.39, 4.46, … **0.07**

Find the next four terms in each arithmetic sequence.

22. −4, −7, −10, −13, … **−16, −19, −22, −25**
23. $\frac{1}{8}, 0, -\frac{1}{8}, -\frac{1}{4}, \ldots$ **$-\frac{3}{8}, -\frac{1}{2}, -\frac{5}{8}, -\frac{3}{4}$**
24. 505, 512, 519, 526, … **533, 540, 547, 554**
25. 1.8, 1.3, 0.8, 0.3, … **−0.2, −0.7, −1.2, −1.7**
26. $\frac{2}{3}, \frac{4}{3}, 2, \frac{8}{3}, \ldots$ **$\frac{10}{3}, 4, \frac{14}{3}, \frac{16}{3}$**
27. −1.1, −0.9, −0.7, −0.5 **−0.3, −0.1, 0.1, 0.3**

Find the given term of each arithmetic sequence.

28. 5, 10, 15, 20, …; 17th term **85**
29. 121, 110, 99, 88, …; 10th term **22**
30. −2, −5, −8, −11, …; 41st term **−122**
31. −30, −22, −14, −6, …; 20th term **122**
32. **Critical Thinking** Is the sequence $5a - 1, 3a - 1, a - 1, -a - 1, \ldots$ arithmetic? If not, explain why not. If so, find the common difference and the next three terms.
Yes (when $a \neq 0$); the common difference is $-2a$; $-3a - 1, -5a - 1, -7a - 1$.

3-6 Exercises

Assignment Guide

Assign *Guided Practice* exercises as necessary.

If you finished Examples **1–3**

Basic 9–38, 41–44
Average 10–14 even, 15–45, 47
Advanced 10–14 even, 15–47

Homework Quick Check
Quickly check key concepts.
Exercises: 10, 14, 15, 20, 30, 36

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them. Exercises 8, 15, 33, 38–39, 41, 47

Construct viable arguments and critique the reasoning of others. Exercises 32, 40

Look for and make use of structure. Exercises 2–7, 9–14, 16–31, 34–37, 45–46

Learn It Online
State Resources Online

MULTI-STEP TEST PREP **Exercise 41** involves using an arithmetic sequence to calculate distance. This exercise prepares students for the Multi-Step Test Prep.

Answers

38a. 1, 1, 2, 3, 5, 8, 13, 21, 34, 55; no; there is no common difference.

b. Yes; every 3rd term is divisible by 2; every 3rd term is the sum of 2 odd numbers.

c. Every 4th term is divisible by 3, and every 5th term is divisible by 5.

40. Possible answer: Find the difference between two successive terms. If the terms of the sequence become greater in value, then the common difference is positive. If the terms of the sequence become lesser in value, then the common difference is negative.

33a. Because you pay $2 per lap, the relationship could be represented by an arithmetic sequence in which $d = 2$.

b. $9, $11, $13, $15; $a_n = 2n + 7$

33. Recreation The rates for a go-cart course are shown.

a. Explain why the relationship described on the flyer could be represented by an arithmetic sequence.

b. Find the cost for 1, 2, 3, and 4 laps. Write a rule to find the nth term of the sequence.

c. How much would 15 laps cost? **$37**

d. What if...? After 9 laps, you get the 10th one free. Will the sequence still be arithmetic? Explain.

No; the sequence will not increase by 2 from the 9th term to the 10th term.

Find the given term of each arithmetic sequence.

34. 2.5, 8.5, 14.5, 20.5, ...; 30th term **176.5**

35. 189.6, 172.3, 155, 137.7, ...; 18th term **−104.5**

36. $\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{7}{4}, \ldots$; 15th term $\frac{29}{4}$

37. $\frac{2}{3}, \frac{11}{12}, \frac{7}{6}, \frac{17}{12}, \ldots$; 25th term $\frac{20}{3}$

LINK Number Theory

Fibonacci numbers occur frequently throughout nature. The number of petals on many flowers are numbers of the Fibonacci sequence. Two petals on a flower are rare but 3, 5, and even 34 petals are common.

39a. $a_n = 6 + 3(n - 1)$

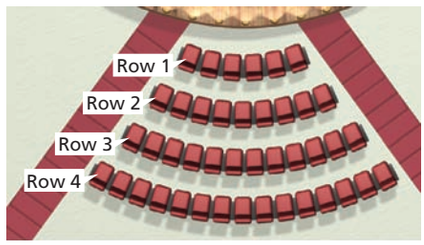
38. Number Theory The sequence 1, 1, 2, 3, 5, 8, 13, … is a famous sequence called the Fibonacci sequence. After the first two terms, each term is the sum of the previous two terms.

a. Write the first 10 terms of the Fibonacci sequence. Is the Fibonacci sequence arithmetic? Explain.

b. Notice that the third term is divisible by 2. Are the 6th and 9th terms also divisible by 2? What conclusion can you draw about every third term? Why is this true?

c. Can you find any other patterns? (*Hint:* Look at every 4th and 5th term.)

39. Entertainment Seats in a concert hall are arranged in the pattern shown.



a. The numbers of seats in the rows form an arithmetic sequence. Write a rule for the arithmetic sequence.

b. How many seats are in the 15th row? **48**

c. A ticket costs $40. Suppose every seat in the first 10 rows is filled. What is the total revenue from those seats? **$7800**

d. What if...? An extra chair is added to each row. Write the new rule for the arithmetic sequence and find the new total revenue from the first 10 rows. **$a_n = 7 + 3(n - 1)$; $8200**

40. Write About It Explain how to find the common difference of an arithmetic sequence. How can you determine whether the arithmetic sequence has a positive common difference or a negative common difference?

MULTI-STEP TEST PREP

41. Juan is traveling to visit universities. He notices mile markers along the road. He records the mile marker every 10 minutes. His father is driving at a constant speed.

a. Copy and complete the table.

b. Write the rule for the sequence. **$a_n = 520 + (n - 1)(-11)$**

c. What does the common difference represent?

d. If this sequence continues, find the mile marker for time interval 10. **421**

Time Interval	Mile Marker
1	520
2	509
3	498
4	**487**
5	**476**
6	**465**

c. number of miles per interval

(t),Digital Vision/gettyimages; (b),cornstock/Getty Photos

PRACTICE A

PRACTICE C

PRACTICE B

Determine whether each sequence is an arithmetic sequence. If so, find the common difference and the next three terms.

1. −10, −7, −4, −1, …
   arithmetic; $d = 3$; 2, 5, 8
2. 0, 1.5, 3, 4.5, …
   arithmetic; $d = 1.5$; 6, 7.5, 9
3. 5, 8, 12, 17, …
   not arithmetic
4. −20, −20.5, −21, −21.5, …
   arithmetic; $d = -0.5$; −22, −22.5, 23

Find the indicated term of each arithmetic sequence.

5. 28th term: 0, −4, −8, −12, …
   −108
6. 15th term: 2, 3.5, 5, 6.5, …
   23
7. 37th term: $a_1 = -3$; $d = 2.8$
   97.8
8. 14th term: $a_1 = 4.2$; $d = -5$
   −60.8
9. 17th term; $a_1 = 2.3$; $d = -2.3$
   −34.5
10. 92nd term; $a_1 = 1$; $d = 0.8$
    73.8
11. A movie rental club charges $4.95 for the first month's rentals. The club charges $18.95 for each additional month. How much is the total cost for one year?
    $213.40
12. A carnival game awards a prize if Kasey can shoot a basket. The charge is $5.00 for the first shot, then $2.00 for each additional shot. Kasey needed 11 shots to win a prize. What is the total amount Kasey spent to win a prize?
    $25.00

READING STRATEGIES

Use the concept map below to help you understand arithmetic sequences.

Arithmetic Sequence

Definition
An arithmetic sequence is a list of numbers whose terms all differ by the same non-zero number.

Formula
The nth term of an arithmetic sequence can be found using $a_n = a_1 + (n - 1)d$ where a_1 is the first term and d is the common difference.

Examples
−2, 1, 4, 7, 10, … (+3 +3 +3 +3)
9, 7.5, 6, 4.5, 3, … (−1.5 −1.5 −1.5 −1.5)

Non-Examples
−4, −3, −1, 2, 6, … (+1 +2 +3 +4)
1, 2, 4, 8, 16, … (×2 ×2 ×2 ×2)

Answer each question.

1. Explain why −4, −3, −1, 2, 6, … is given as a non-example of an arithmetic sequence.
   The terms do not all differ by the same number.
2. Explain why 1, 2, 4, 8, 16, … is given as a non-example of an arithmetic sequence.
   The same number is being multiplied, not added, to each term.
3. Give an example of an arithmetic sequence whose common difference is 5.
   Possible answer: 1, 6, 11, 16, 21, …
4. Find the common difference and the next 3 terms of the sequence 8, $7\frac{1}{2}$, 7, $6\frac{1}{2}$, 6, … .
   $-\frac{1}{2}$; $5\frac{1}{2}$, 5, $4\frac{1}{2}$

Find the indicated term of each arithmetic sequence using the formula given above.

5. 25th term: −14, −8, −2, 4, 10, …
   130
6. 18th term: 122, 120, 118, 116, 114, …
   88

RETEACH

An **arithmetic sequence** is a list of numbers (or **terms**) with a **common difference** between each number. After you find the common difference, you can use it to continue the sequence.

Determine whether each sequence is an arithmetic sequence. If so, find the common difference and the next three terms.

1, 2, 4, 8, … (+1 +2 +4) Find how much you add or subtract to move from term to term.
The difference between terms is *not* constant.
This sequence is *not* an arithmetic sequence.

0, 6, 12, 18, … (+6 +6 +6) Find how much you add or subtract to move from term to term.
The difference between terms is constant.
This sequence is an arithmetic sequence with a common difference of 6.

0, 6, 12, 18, 24, 30, 36 (+6 +6 +6) Use the difference of 6 to find three more terms.

Fill in the blanks with the differences between terms. State whether each sequence is an arithmetic sequence.

1. 14, 12, 10, 8, …
   −2 −2 −2
   Is this an arithmetic sequence? yes
2. 0.3, 0.6, 1.0, 1.5, …
   +0.3 +0.4 +0.5
   Is this an arithmetic sequence? no

Use the common difference to find the next three terms in each arithmetic sequence.

3. 7, 4, 1, −2, −5, −8, −11, … (−3 −3 −3 −3 −3 −3)
4. −5, 0, 5, 10, 15, 20, 25, … (+5 +5 +5)

Determine whether each sequence is an arithmetic sequence. If so, find the common difference and the next three terms.

5. −1, 2, −3, 4, …
   no
6. 1.25, 3.75, 6.25, 8.75, …
   yes; 2.5; 11.25, 13.75, 16.25

TEST PREP

42. What are the next three terms in the arithmetic sequence −21, −12, −3, 6, … ?

Ⓐ 9, 12, 15 Ⓑ 15, 24, 33 Ⓒ 12, 21, 27 Ⓓ 13, 20, 27

43. What is the common difference for the data listed in the second column?

Ⓕ −1.8 Ⓗ 2.8

Ⓖ 1.8 Ⓙ −3.6

Altitude (ft)	Boiling Point of Water (°F)
1000	210.2
2000	208.4
3000	206.6

44. Which of the following sequences is NOT arithmetic?

Ⓐ −4, 2, 8, 14, … Ⓑ 9, 4, −1, −6, … Ⓒ 2, 4, 8, 16, … Ⓓ $\frac{1}{3}, 1\frac{1}{3}, 2\frac{1}{3}, 3\frac{1}{3}, \ldots$

CHALLENGE AND EXTEND

45. The first term of an arithmetic sequence is 2, and the common difference is 9. Find two consecutive terms of the sequence that have a sum of 355. What positions in the sequence are the terms? **173 and 182; 20th and 21st terms**

46. The 60th term of an arithmetic sequence is 106.5, and the common difference is 1.5. What is the first term of the sequence? **18**

47. **Athletics** Verona is training for a marathon. The first part of her training schedule is shown below.

Session	1	2	3	4	5	6
Distance Run (mi)	3.5	5	6.5	8	9.5	11

Session 16; yes; she increases the amount she runs by 1.5 miles each time.

a. If Verona continues this pattern, during which training session will she run 26 miles? Is her training schedule an arithmetic sequence? Explain.

b. If Verona's training schedule starts on a Monday and she runs every third day, on which day will she run 26 miles? **Thursday**

TEST PREP DOCTOR In **Exercise 43,** tell students to note that the data in the second column are decreasing. This means that the common difference will be negative. They can eliminate choices **G** and **H.** Students who chose **J** may not understand what the common difference is. Remind them that d is the difference between consecutive terms.

In **Exercise 44,** point out to students that although choice **C** does have a pattern, it is not arithmetic. You cannot add the same number to each term to get the next term.

Journal

Have students compare and contrast determining whether a sequence is arithmetic to determining whether a function is linear.

ALTERNATIVE ASSESSMENT

Have students create three arithmetic sequences. Then have them find the 10th, 25th, and 100th terms for each sequence.

Power Presentations with PowerPoint®

3-6 Lesson Quiz

Determine whether each sequence appears to be an arithmetic sequence. If so, find the common difference and the next three terms.

1. 3, 9, 27, 81, . . . no

2. 5, 6.5, 8, 9.5, . . . yes; 1.5; 11, 12.5, 14

Find the indicated term of each arithmetic sequence.

3. 23rd term: −4, −7, −10, −13, . . . −70

4. 40th term: 2, 7, 12, 17, . . . 197

5. 7th term: $a_1 = -12$, $d = 2$ 0

6. 34th term: $a_1 = 3.2$, $d = 2.6$ 89

7. On day 1, Zelle has knitted 61 rows of a scarf. Each following day she adds 17 more rows. How many total rows has Zelle knitted on day 16? 316 rows

Also available online

PROBLEM SOLVING

Find the indicated term of each arithmetic sequence.

1. Darnell has a job and his saving his paychecks each week.

Weeks	1	2	3	4
Savings	$130	$260	$390	$520

How much will Darnell have saved after 11 weeks?
$1430

2. A tube containing 3 ounces of toothpaste is being used at a rate of 0.15 ounces per day. How much toothpaste will be in the tube after one week?
1.95 ounces

3. A new car costs $13,000 and is depreciating by $900 each year. How much will the car be worth after 4 years?
$9400

4. Jessie is playing an arcade game that costs 50¢ for the first game and 25¢ to continue if she loses. How much will she spend on the game if she continues 9 times?
$2.75

Use the graph below to answer questions 5–9. The graph shows the size of Ivor's ant colony over the first four weeks. Assume the ant population will continue to grow at the same rate. Select the best answer.

5. Which of the following shows how many ants Ivor will have in the next three weeks?
A 315, 341, 367
B 317, 343, 369
C 318, 334, 350
D 319, 345, 371

6. Which rule can be used to find how large the colony will be in n weeks?
F $a_n = 215 + 26n$
G $a_n = 215n + 26$
H $a_n = 215(n - 1) + 26$
J $a_n = 215 + 26(n - 1)$

7. How many ants will Ivor have in 27 weeks?
A 891 C 5616
B 917 D 5831

8. Ivor's ants weigh 1.5 grams each. How many grams do all of his ants weigh in 13 weeks?
F 660.5 H 722
G 683 **J** 790.5

9. When the colony reaches 1385 ants, Ivor's ant farm will not be big enough for all of them. In how many weeks will the ant population be too large?
A 45 C 47
B 46 D 48

CHALLENGE

The sequence of numbers below is known as the sequence of whole numbers. At an early age, children learn to count by using whole numbers.

0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, ...

When you divide each whole number by a fixed number of your choice, you can create a new sequence. The diagram at the right will help you recall the meaning of the terms related to division.

In Exercises 1–6, consider the set of whole numbers. Write the first 10 terms of the sequence defined by the specified division.

1. the remainder you get when you divide by 1 **0, 0, 0, 0, . . .**
2. the remainder you get when you divide by 2 **0, 1, 0, 1, 0, . . .**
3. the remainder you get when you divide by 3 **0, 1, 2, 0, 1, 2, . . .**
4. the remainder you get when you divide by 4 **0, 1, 2, 3, 0, 1, 2, 3, . . .**
5. the remainder you get when you divide by 5 **0, 1, 2, 3, 4, 0, 1, 2, 3, 4, . . .**
6. the remainder you get when you divide by 6 **0, 1, 2, 3, 4, 5, 0, 1, 2, 3, 4, 5, . . .**
7. a. Suppose that you divide each of the whole numbers by 12. Without writing any division or remainders, write a conjecture about the sequence of remainders that you think you will get.
The sequence will range from 0 to 11 and then repeat the pattern from 0 to 11. 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, . . .
b. Write a brief justification of your conjecture from part **a.**
The remainder will always be a whole number ranging from 0 to 1 less than the divisor. The remainders repeat because every whole number is either a multiple of the divisor or a multiple of the divisor plus a whole number between 1 and 1 less than the divisor inclusive.
8. a. The following sequence of remainders is the result of dividing each whole number by an unknown number. By what number are the whole numbers being divided?
0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, ...
10
b. Write a brief justification of your conjecture from part **a.**
Every whole number divided by 10 will give a remainder that is a whole number from 0 to 9 inclusive.
9. It is noon. Use division and remainders to find what time will be 99 hours from now.
3:00 P.M.

SECTION 3B

MULTI-STEP TEST PREP

Organizer

Objective: Assess students' ability to apply concepts and skills in a real-world format.

Online Edition

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them.

Learn It Online
State Resources Online

STATE

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them.

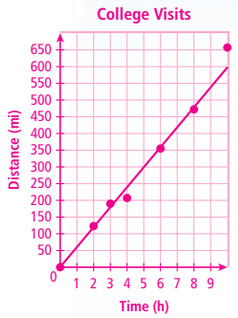
Applying Functions

College Knowledge Myra is helping her brother plan a college visit 10 hours away from their home. She creates a table listing approximate travel times and distances from their home.

Time (h)	Distance (mi)
0	0
2	123
3	190
4	207
6	355
8	472
10	657

1. Create a scatter plot for the data.
2. Draw a trend line through the data.
3. Based on the trend line, how many miles will they have traveled after 5 hours? Possible answer: 300 miles
4. If Myra's brother decided to visit a college 13 hours away from their home, approximately how many miles will they travel? approximately 800 miles
5. To find the average speed for the entire trip, find $\frac{\text{change in distance}}{\text{change in time}}$ between the initial ordered pair and the final ordered pair. Include the units. 65.7 mi/h

1, 2.



(tl; comstock/Getty Photos; (bl); AP Photo/Daniel Hulshizer

INTERVENTION

Scaffolding Questions

1. What are two points that will be graphed? Possible answer: (0, 0), (4, 207)
2. Does the trend line have to go through all the points? no

3–4. How do you use a trend line to make predictions? Find the point on the graph with the given *x*-value and use it to identify the corresponding *y*-value.

5. What is the total change in distance? 657 mi What is the total change in time? 10 h

Extension

Based on the scatter plot, how far away would you estimate the college to be if Myra and her brother traveled for 15 hours? about 985 mi

READY TO GO ON?

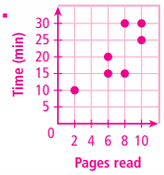
Quiz for Lessons 3-5 Through 3-6

3-5 Scatter Plots and Trend Lines

The table shows the time it takes different people to read a given number of pages.

Pages Read	2	6	6	8	8	10	10
Time (min)	10	15	20	15	30	25	30

1. Graph a scatter plot using the given data.



2. Describe the correlation illustrated by the scatter plot. positive

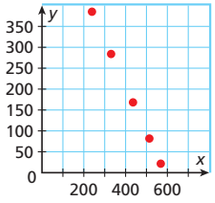
Choose the scatter plot that best represents the described relationship. Explain.

3. number of movie tickets sold and number of available seats A

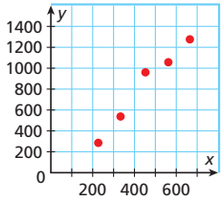
4. number of movie tickets sold and amount of concession sales B

5. number of movie tickets sold and length of movie C

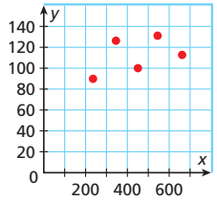
Graph A



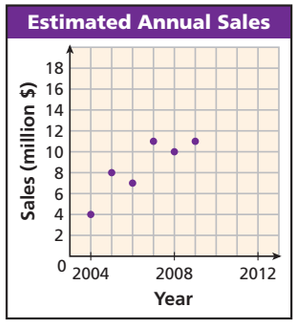
Graph B



Graph C



6. The scatter plot shows the estimated annual sales for an electronics and appliance chain of stores for the years 2004–2009. Based on this relationship, predict the annual sales in 2012.
Possible answer: 16.5 million



3-6 Arithmetic Sequences

Determine whether each sequence appears to be an arithmetic sequence. If so, find the common difference and the next three terms.

7. 7, 3, −1, −5, … yes; −4; −9, −13, −17

8. 3, 6, 12, 24, … no

9. −3.5, −2, −0.5, 1, … yes; 1.5; 2.5, 4, 5.5

Find the indicated term of the arithmetic sequence.

10. 31st term: 12, 7, 2, −3, … −138

11. 22nd term: $a_1 = 6$; $d = 4$ 90

12. With no air resistance, an object would fall 16 feet during the first second, 48 feet during the second second, 80 feet during the third second, 112 feet during the fourth second, and so on. How many feet will the object fall during the ninth second? 272 ft

READY TO GO ON? SECTION 3B

Organizer

Objective: Assess students' mastery of concepts and skills in this section.

Resources

Assessment Resources
Section Quiz

INTERVENTION

Resources

***Ready to Go On? Intervention and Enrichment* Worksheets**

***Ready to Go On?* Online**

READY TO GO ON?
Diagnose and Prescribe

NO INTERVENE

Ready to Go On? Intervention	*Ready to Go On? Intervention*, Section 3B: Worksheets	Online	
Lesson 3-5	3-5 Intervention	Activity 3-5	Diagnose and Prescribe Online
Lesson 3-6	3-6 Intervention	Activity 3-6	

YES ENRICH

***Ready to Go On? Enrichment*, Section 3B**
Worksheets
Online

Organizer

Objective: Help students organize and review key concepts and skills presented in this chapter.

Online Edition
Multilingual Glossary

Countdown

Resources

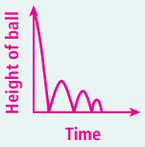
Learn It Online
Multilingual Glossary

Lesson Tutorial Videos

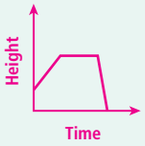
Answers

1. domain
2. negative correlation
3. term
4. continuous

5. continuous



6. continuous



Vocabulary

arithmetic sequence	**function**	**range**
common difference	**function notation**	**relation**
continuous graph	**function rule**	**scatter plot**
correlation	**independent variable**	**sequence**
dependent variable	**negative correlation**	**term**
discrete graph	**no correlation**	**trend line**
domain	**positive correlation**	

Complete the sentences below with vocabulary words from the list above.

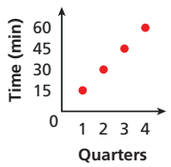
1. The set of x-coordinates of the ordered pairs of a relation is called the ___?___.
2. If one set of data values increases as another set of data values decreases, the relationship can be described as having a(n) ___?___.
3. A sequence is an ordered list of numbers where each number is a(n) ___?___.

3-1 Graphing Relationships

EXAMPLES

Sketch a graph for each situation. Tell whether the graph is continuous or discrete.

- A parking meter has a limit of 1 hour. The cost is \$0.25 per 15 minutes and the meter accepts quarters only.

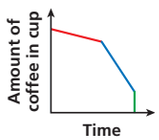


Since only quarters are accepted, the points are not connected.

The graph is discrete.

- Ian bought a cup of coffee. At first, he sipped slowly. As it cooled, he drank more quickly. The last bit was cold, and he dumped it out.

As time passes the coffee was **sipped slowly,** **drank more quickly,** and then **dumped out.**



The graph is continuous.

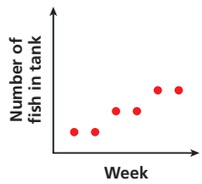
EXERCISES

Sketch a graph for each situation. Tell whether the graph is continuous or discrete.

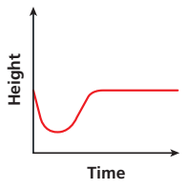
4. A girl was walking home at a steady pace. Then she stopped to talk to a friend. After her friend left, she jogged the rest of the way home.
5. A ball is dropped from a second story window and bounces to a stop on the patio below.
6. Jason was on the second floor when he got a call to attend a meeting on the sixth floor. He took the stairs. After the meeting, he took the elevator to the first floor.

Write a possible situation for each graph.

7.



8.



7. Possible answer: A family buys a fish tank and some fish. After two weeks, they buy some more fish. After two more weeks, they even buy more fish.
8. Possible answer: A monkey swings from a high branch to a lower branch. He climbs along the branch. Then he jumps to a higher branch and takes a nap.

3-2 Relations and Functions

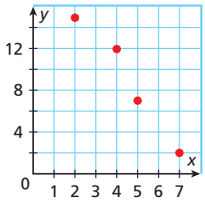
EXAMPLES

■ **Express the relation $\{(2, 15), (4, 12), (5, 7), (7, 2)\}$ as a table, as a graph, and as a mapping diagram.**

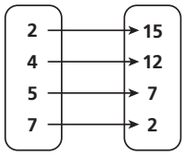
Table

x	y
2	15
4	12
5	7
7	2

Graph



Mapping Diagram



Give the domain and range of each relation. Tell whether the relation is a function. Explain.

■

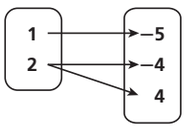
x	y
−3	0
−2	0
−1	1

D: $\{-3, -2, -1\}$

R: $\{0, 1\}$

The relation is a function because each domain value is paired with exactly one range value.

■

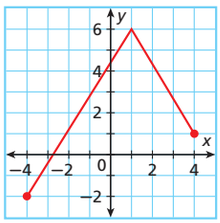


D: $\{1, 2\}$

R: $\{-5, -4, 4\}$

The relation is not a function because one domain value is paired with two range values.

■



D: $-4 \le x \le 4$

R: $-2 \le y \le 6$

The relation is a function because every x-value is paired with exactly one y-value.

EXERCISES

Express each relation as a table, as a graph, and as a mapping diagram.

9. $\{(-1, 0), (0, 1), (2, 1)\}$

10. $\{(-2, -1), (-1, 1), (2, 3), (3, 4)\}$

Give the domain and range of each relation.

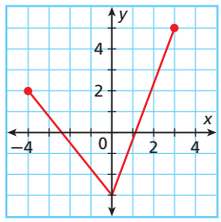
11. $\{(-4, 5), (-2, 3), (0, 1), (2, -1)\}$

12. $\{(-2, -1)\ (-1, 0), (0, -1), (1, 0), (2, -1)\}$

13.

x	0	1	4	1	4
y	0	−1	−2	1	2

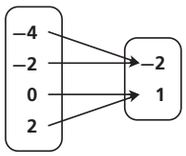
14.



Give the domain and range of each relation. Tell whether the relation is a function. Explain.

15. $\{(-5, -3), (-3, -2), (-1, -1), (1, 0)\}$

16.



17.

x	1	2	3	4	1
y	3	2	1	0	−1

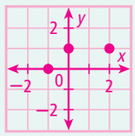
18. A local parking garage charges \$5.00 for the first hour plus \$1.50 for each additional hour or part of an hour. Write a relation as a set of ordered pairs in which the x-value represents the number of hours and the y-value represents the cost for x hours. Use a domain of 1, 2, 3, 4, 5. Is this relation a function? Explain.

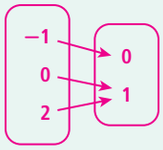
19. A baseball coach is taking the team for ice cream. Four students can ride in each car. Create a mapping diagram to show the number of cars needed to transport 8, 10, 14, and 16 students. Is this relation a function? Explain.

Answers

9.

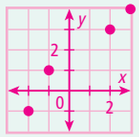
x	−1	0	2
y	0	1	1

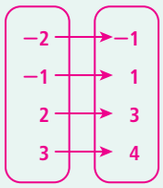




10.

x	−2	−1	2	3
y	−1	1	3	4





11. D: $\{-4, -2, 0, 2\}$; R: $\{-1, 1, 3, 5\}$

12. D: $\{-2, -1, 0, 1, 2\}$; R: $\{-1, 0\}$

13. D: $\{0, 1, 4\}$; R: $\{-2, -1, 0, 1, 2\}$

14. D: $-4 \le x \le 3$; R: $-3 \le y \le 5$

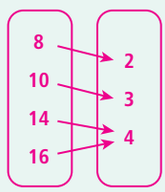
15. D: $\{-5, -3, -1, 1\}$; R: $\{-3, -2, -1, 0\}$; yes; every element of the domain is assigned to exactly one element in the range.

16. D: $\{-4, -2, 0, 2\}$; R: $\{-2, 1\}$; yes; each element of the domain is assigned to exactly one element in the range.

17. D: $\{1, 2, 3, 4\}$; R: $\{-1, 0, 1, 2, 3\}$; no; the x-value 1 is assigned to the y-value 3 and the y-value -1.

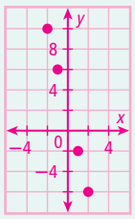
18. $\{(1, 5.00), (2, 6.50), (3, 8.00), (4, 9.50), (5, 11.00)\}$; yes; each x-value has exactly one y-value.

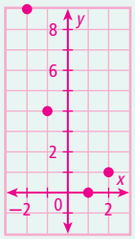
19.

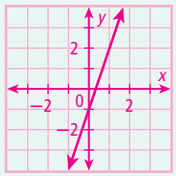


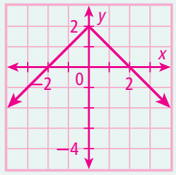
Yes; each element in the domain is assigned to exactly one element in the range.

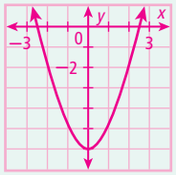
Answers

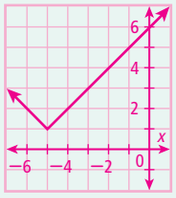
20. y is 7 less than x; $y = x - 7$.
21. y is 9 times x; $y = 9x$.
22. ind.: number of cakes; dep.: cost; $f(c) = 6c$
23. ind.: number of CDs Raul will buy; dep.: number of CDs Tim will buy; $g(n) = 2n$
24. 14
25. −11
26. 6; −1
27. 

28. 

29. 

30. 

31. 

32. 

3-3 Writing Functions

EXAMPLES

■ **Determine a relationship between the x- and y-values in the table. Write an equation.**

x	1	2	3	4
y	−3	−6	−9	−12

What are possible relationships between the x-values and the y-values?

$1 - 4 = -3$ $\quad 1(-3) = -3$

$2 - 4 \neq -6$ ✗ $\quad 2(-3) = -6$ ✓

$3(-3) = -9$ ✓

$4(-3) = -12$ ✓

$y = -3x$ *Write an equation.*

■ **Nia earns $5.25 per hour. Identify the independent and dependent variables. Write an equation in function notation for the situation.**

Nia's **pay** depends on the **number of hours** she works.
Dependent: **pay**
Independent: **hours**
Let h represent the number of hours Nia works.
The function for Nia's pay is $f(h) = 5.25h$.

EXERCISES

Determine the relationship between the x- and y-values. Write an equation.

20.

x	1	2	3	4
y	−6	−5	−4	−3

21. $\{(1, 9), (2, 18), (3, 27), (4, 36)\}$

Identify the independent and dependent variables. Write an equation in function notation for the situation.

22. A baker spends $6 on ingredients for each cake he bakes.
23. Tim will buy twice as many CDs as Raul.

Evaluate each function for the given input values.

24. For $f(x) = -2x + 4$, find $f(x)$ when $x = -5$.
25. For $g(n) = -n^2 - 2$, find $g(n)$ when $n = -3$.
26. For $h(t) = 7 - |t + 3|$, find $h(t)$ when $t = -4$ and when $t = 5$.

3-4 Graphing Functions

EXAMPLE

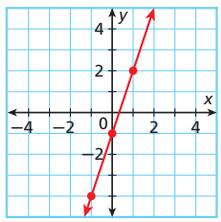
■ **Graph the function $y = 3x - 1$.**

Step 1 Choose several values of x to generate ordered pairs.

x	$y = 3x - 1$	y
−1	$y = 3(-1) - 1 = -4$	−4
0	$y = 3(0) - 1 = -1$	−1
1	$y = 3(1) - 1 = 2$	2
2	$y = 3(2) - 1 = 5$	5

Step 2 Plot enough points to see a pattern.

Step 3 Draw a line through the points to show all the ordered pairs that satisfy this function.



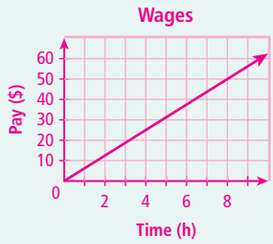
EXERCISES

Graph each function for the domain $\{-2, -1, 1, 2\}$.

27. $4x + y = 2$
28. $y = (1 - x)^2$

Graph each function.

29. $3x - y = 1$
30. $y = 2 - |x|$
31. $y = x^2 - 6$
32. $y = |x + 5| + 1$
33. The function $y = 6.25x$ describes the amount of money y Peter gets paid after x hours. Graph the function. Use the graph to estimate how much money Peter gets paid after 7 hours.

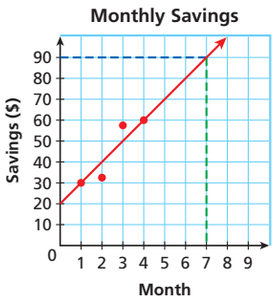
33. 

Possible answer: about $44

3-5 Scatter Plots and Trend Lines

EXAMPLE

■ **The graph shows the amount of money in a savings account. Based on this relationship, predict how much money will be in the account in month 7.**



Draw a line that has about the same number of points above and below it. Your line may or may not go through data points.

Find the point on the line whose x-value is 7.

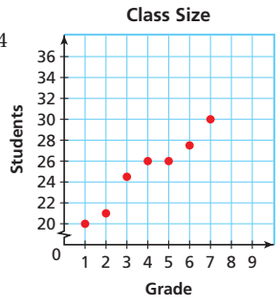
Based on the data, $90 is a reasonable prediction.

EXERCISES

34. The table shows the value of a car for the given years. Graph a scatter plot using the given data. Describe the correlation illustrated by the scatter plot.

Year	2000	2001	2002	2003
Value (thousand $)	28	25	23	20

35. The graph shows the results of a 2003–2004 survey on class size at the given grade levels. Based on this relationship, predict the class size for the 9th grade.



3-6 Arithmetic Sequences

EXAMPLES

■ **Determine whether the sequence appears to be arithmetic. If so, find the common difference and the next three terms.**

−8, −5 , −2 , 1,...

Step 1 Find the difference between successive terms.

−8, −5, −2, 1,... *The common difference is 3.*

+3 +3 +3

Step 2 Use the common difference to find the next 3 terms.

−8, −5, −2, 1, **4**, **7**, **10**

+3 +3 +3

■ **Find the 18th term of the arithmetic sequence for which $a_1 = -4$ and $d = 6$.**

$a_n = a_1 + (n - 1)d$ *Write the rule.*

$a_{18} = -4 + (18 - 1)6$ *Substitute.*

$= -4 + (17)6$ *Simplify.*

$= -4 + 102$ *Simplify.*

$= 98$

The 18th term is 98.

EXERCISES

Determine whether each sequence appears to be arithmetic. If so, find the common difference and the next three terms.

36. 20, 14, 8, 2,...

37. −15, −12, −9, −4,...

38. 5, 4, 2, −1,...

39. −8, −5.5, −3, −0.5,...

Find the indicated term of each arithmetic sequence.

40. 31st term: −15, −11, −7, −3,...

41. 24th term: $a_1 = 7$; $d = -3$

42. 17th term: $a_1 = -20$; $d = 2.5$

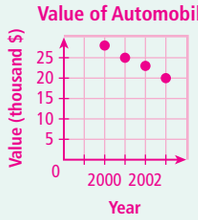
43. Marie has $180 in a savings account in week 1. She plans to deposit $12 each following week. Assuming that she does not withdraw any money from her account, what will her balance be in week 20?

44. The table shows the temperature at the given heights above sea level. Use an arithmetic sequence to find the temperature at 8000 feet above sea level.

Height Above Sea Level (thousand feet)	1	2	3	4
Temperature (°C)	30	23.5	17	10.5

Answers

34. Value of Automobile



Negative; as the time increases, the value of the automobile decreases.

35. Possible answer: 33

36. yes; −6; −4, −10, −16

37. no

38. no

39. yes; 2.5; 2, 4.5, 7

40. 105

41. −62

42. 20

43. $408

44. −15.5 °C

CHAPTER 3

CHAPTER TEST

Organizer

Objective: Assess students' mastery of concepts and skills in this chapter.

INTERACTIVE **Online Edition**

Resources

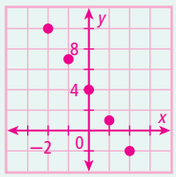
Assessment Resources

Chapter Tests

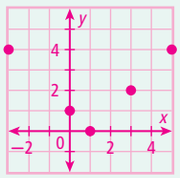
- Free Response (Levels A, B, C)
- Multiple Choice (Levels A, B, C)
- Performance Assessment

Answers

9.



10.



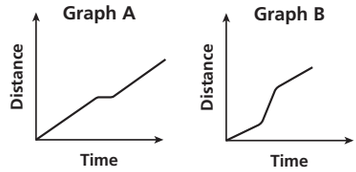
Learn It Online
State Resources Online

CHAPTER 3

CHAPTER TEST

Choose the graph that best represents each situation.

1. A person walks leisurely, stops, and then continues walking. **A**
2. A person jogs, then runs, and then jogs again. **B**



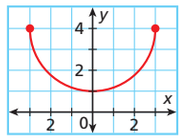
Give the domain and range for each relation. Tell whether the relation is a function. Explain.

3.

x	−2	1	0	1	3
y	3	2	1	0	−1

D: {−2, 1, 0, 3}; R: {−1, 0, 1, 2, 3}; no; the *x*-value 1 is assigned to the *y*-value 2 and the *y*-value 0.

4.



D: $-3 \le x \le 3$; R: $1 \le y \le 4$; yes; each element in the domain is assigned to exactly one element in the range.

5. Bowling costs \$3 per game plus \$2.50 for shoe rental. Identify the independent and dependent variables. Write an equation in function notation for the situation.
ind.: games played; dep.: total cost; $y = 3x + 2.50$

Evaluate each function for the given input values.

6. For $f(x) = -3x + 4$, find $f(x)$ when $x = -2$. **10**
7. For $f(x) = 2x^2$, find $f(x)$ when $x = -3$. **18**
8. An engraver charges a \$10 fee plus \$6 for each line of engraving. Write a function to describe the situation. Find a reasonable domain and range for the function for up to 8 lines.
$y = 10 + 6x$; D: {1, 2, 3, 4, 5, 6, 7, 8}; R: {16, 22, 28, 34, 40, 46, 52, 58}

Graph each function for the given domain.

9. $3x + y = 4$; D: $\{-2, -1, 0, 1, 2\}$
10. $y = |x - 1|$; D: $\{-3, 0, 1, 3, 5\}$
11. $y = x^2 - 1$; D: $\{-2, -1, 0, 1, 2\}$

Graph each function.

12. $y = x - 5$
13. $y = x^2 - 5$
14. $y = |x| + 3$
15. The function $y = 30x$ describes the amount of interest y earned in a savings account in x years. Graph the function. Use the graph to estimate the total amount of interest earned in 7 years.
17. Negative; as the child's age increases, the number of hours of sleep needed decreases.

The table shows possible recommendations for the amount of sleep that children should get every day.

Age (yr)	1	2	3	4	5	14
Sleep Needed (h)	14	13	12	12	11	9

16. Graph a scatter plot of the given data.
17. Describe the correlation illustrated by the scatter plot.
18. Predict how many hours of sleep a 16-year-old needs. **Possible answer: 7 h**

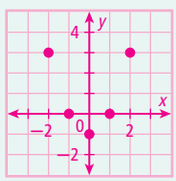
Determine whether each sequence appears to be an arithmetic sequence. If so, find the common difference and the next three terms.

19. 11, 6, 1, −4,... **yes; −5; −9, −14, −19**
20. −4, −3, −1, 2,... **no**
21. 7, 21, 30, 45,... **no**

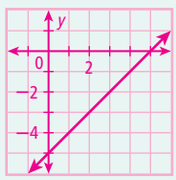
Find the indicated term of the arithmetic sequence.

22. 32nd term: 18, 11, 4, −3,... **−199**
23. 24th term: $a_1 = 4$; $d = 6$ **142**
24. Mandy's new job has a starting salary of \$16,000 and annual increases of \$800. How much will she earn during her fifth year? **\$19,200**

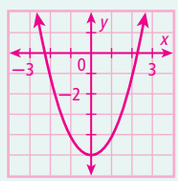
11.



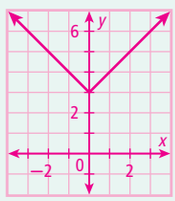
12.



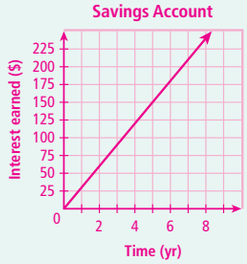
13.



14.



15. Possible answer: \$210



16. See Additional Answers.

COLLEGE ENTRANCE EXAM PRACTICE

CHAPTER 3

FOCUS ON ACT

Questions on the ACT Mathematics Test do not require the use of a calculator, but you may bring one to use with the test. Make sure that it is a calculator that is on the approved list for the ACT.

You may want to time yourself as you take this practice test. It should take you about 6 minutes to complete.

HOT TIP! When taking the test, you will be more comfortable using a calculator that you are used to. If you already have a calculator, make sure it is one of the permitted calculators. If you plan to use a new one, make sure to practice using it before the test.

1. The soccer team is ordering new uniforms. There is a one-time setup charge of $50.00, and each uniform costs $23.50. Which of the following best describes the total cost C for ordering uniforms for p players?

(A) $C = 23.50p$

(B) $C = 50p$

(C) $C = 73.50p$

(D) $C = 23.50p + 50$

(E) $C = 50p + 23.50$

2. In the given relation, what domain value corresponds to the range value -2?
$\{(-1, 2), (-2, 4), (2, 5), (0, -2), (2, 0)\}$

(F) -2

(G) 0

(H) 2

(J) 4

(K) 5

3. Evaluate $h(x) = \frac{1}{2}(5 - 6x) + 9x$ when $x = \frac{2}{3}$.

(A) $\frac{9}{2}$

(B) $\frac{13}{2}$

(C) 7

(D) $\frac{19}{2}$

(E) $\frac{23}{2}$

4. What is the seventh term of the arithmetic sequence $-4, -1, 2 \ldots$?

(F) 5

(G) 10

(H) 11

(J) 14

(K) 17

5. The graph of which function is shown below?

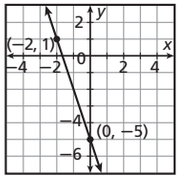
(A) $y = -3x - 5$

(B) $y = -\frac{1}{3}x - \frac{5}{3}$

(C) $y = -5x - 3$

(D) $y = 3x - 5$

(E) $y = 5x + 3$



6. Which of the following relations is NOT a function?

(F) $\{(0, 1), (1, 2), (2, 3), (3, 4)\}$

(G) $\{(1, 2), (2, 2), (3, 3), (4, 3)\}$

(H) $\{(0, 2), (2, 4), (4, 1), (1, 3)\}$

(J) $\{(1, 3), (4, 2), (2, 0), (3, 4)\}$

(K) $\{(0, 2), (1, 3), (4, 3), (1, 2)\}$

Organizer

Objective: Provide practice for college entrance exams such as the ACT.

INTERACTIVE **Online Edition**

Resources

College Entrance Exam Practice

Questions on the ACT represent the following content areas:

Pre-Algebra, 23%

Elementary Algebra, 17%

Intermediate Algebra, 15%

Coordinate Geometry, 15%

Plane Geometry, 23%

Trigonometry, 7%

Items on this page focus on:

- Elementary Algebra
- Coordinate Geometry

TEST PREP DOCTOR

1. Students who chose **A** did not take into account the setup charge for the order. Students who chose **C** added the one-time setup charge to the price for each uniform.

2. Students who chose **J** may not understand the meaning of the terms *domain* and *range*. Students who selected a different incorrect answer may not have read the problem carefully.

3. Students who chose **E** found $h\left(\frac{3}{2}\right)$ instead of $h\left(\frac{2}{3}\right)$. Students who chose **A** may have tried to simplify the expression before evaluating it and may not have applied the distributive property correctly.

4. Students who chose **G** may have incorrectly remembered the formula for finding the nth term of an arithmetic sequence as $a_n = a_1 + (d - 1)n$. Suggest to students that when the term to be found is a low-numbered term of the sequence, they can add the common difference repeatedly to find the result.

5. Students who chose **B** reversed the values of x and y in the definition of the function.

6. Students who chose **G** may think that a function is a relation in which each element of the range corresponds to exactly one element of the domain.

CHAPTER 3 TEST TACKLER

Organizer

Objective: Provide opportunities to learn and practice common test-taking strategies.

INTERACTIVE **Online Edition**

TEST PREP DOCTOR

This Test Tackler explains how extended-response test items are scored and demonstrates how to create a response that is deserving of full credit. Explain to students that extended-response questions are longer and more complex than short-response questions and that the scoring rubric used is based on a 4-point scale rather than a 2-point scale. Use the analogy that these questions are like essay questions in an English class; they require complete explanations to receive full credit.

Extended Response: Understand the Scores

Extended response test items are typically multipart questions that require a high level of thinking. The responses are scored using a 4-point rubric. To receive full credit, you must correctly answer all parts of the question and provide a clear explanation. A partial answer is worth 2 or 3 points, an incorrect solution is worth 1 point, and no response is worth 0 points.

EXAMPLE 1

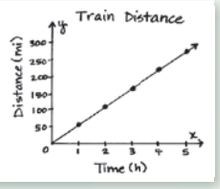
Extended Response A train traveling from Boston, Massachusetts, to Richmond, Virginia, averages about 55 miles per hour. Define variables, write an equation, make a table, and draw a graph to show the distance the train travels in 5 hours.

Here are examples of four different responses and their scores using the rubric shown.

4-point response:

Let d = the distance the train travels.
Let t = the time the train travels.
d = 55t

Time (h)	0	1	2	3	4	5
Distance (mi)	0	55	110	165	220	275

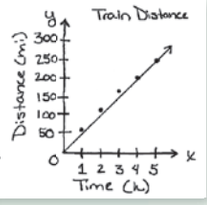


3-point response:

Let d = the distance the train travels
Let t = the time the train travels

d = 55t

Time (h)	0	1	2	3	4	5
Distance (mi)	0	55	110	165	200	250

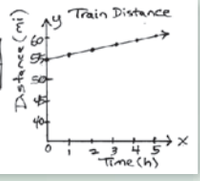


The student shows all of the work, but there are two minor computation errors when t = 4 and t = 5.

2-point response:

d = 55 + t

Time (h)	0	1	2	3	4	5
Distance (mi)	55	56	57	58	59	60



The student writes an incorrect equation and uses it to create an incorrect table and graph.

1-point response:

d = 55t

The student does not answer two parts of the question.

TEST PREP DOCTOR

As you read through each test item, ask students to determine how many parts are in each question. Encourage students to underline key words in the question: *List, Explain, Solve*, etc.

HOT TIP! Never leave an extended-response test item blank. At least try to define variables or write equations where appropriate. You will get some points just for trying.

Read each test item and answer the questions that follow using the rubric below.

Scoring Rubric:

4 points: The student shows all of the work, correctly answers all parts of the question, and provides a clear explanation.

3 points: The student shows most of the work and provides a clear explanation but has a minor computation error, or the student shows all of the work and arrives at a correct solution but does not provide a clear explanation.

2 points: The student makes major errors resulting in an incorrect solution, or the student gives a correct solution but does not show any work nor provide an explanation.

1 point: The student shows no work and gives an incorrect solution.

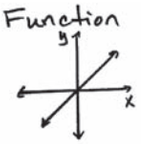
0 points: The student gives no response.

Item A

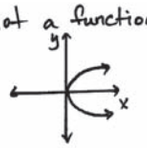
Extended Response Draw a graph that is a function. Explain why it is a function. Then draw a graph that is NOT a function. Explain why it is not a function.

1. What should be included in a 4-point response?
2. Explain how would you score the response below.

Function



Not a function



The first graph is a function because each x-value has exactly one y-value. When x=1, y=1. The second graph is not a function because there is more than one y-value for each x-value. When x=1, y=1, and y=−1. Therefore, the second graph is not a function.

Item B

Extended Response A car travels at a steady rate of 60 miles per hour. Identify the independent and dependent variables. Describe the domain and range. Write an equation to describe the situation.

3. Ana wrote the response below.

The equation is $y = 60x$. The independent variable is time and the dependent variable is distance. The domain and range are all real numbers.

Explain how would you score Ana's response.

4. If you did not give Ana full credit, what should be added to Ana's response, if anything, so that it receives full credit?

Item C

Extended Response Lara bought 8 notebooks and 4 binders. She spent $14 total without tax. How much did each notebook cost if each binder cost $2.50? Write an equation and find the solution.

5. Explain how would you score the response below.

Let s = the cost of each notebook.
Let b = the cost of each binder.
$8s + 4b = 14$
$8s + 4(2.50) = 14$
$8s + 10 = 14$
$8s = 4$
$s = 2$ *The notebooks cost $2 each*

6. If you did not give the response full credit, what should be added to the response, if anything, so that it receives full credit?

Answers

1. Possible answer: a graph of a function and an explanation of why it is a function; a graph of a relation that is not a function and an explanation of why it is not a function
2. Possible answer: The response is worth 4 points because it has two graphs, one of which is a function and one of which is not a function. The explanations are complete and clearly written.
3. Possible answer: 3; Ana answered 2 of the sections correctly, but the domain and range are not all real numbers.
4. The domain and range should be all real numbers greater than or equal to zero.
5. Possible answer: The response is worth 3 points because the student showed all of the work and completed all parts of the question, but in the last step of the equation, the student divided incorrectly.
6. Change the last step to $s = 0.5$ and then state that the notebooks cost $0.50.

Answers to Test Items

A. See answer to Problem 2.

B. The equation is $y = 60x$. The independent variable is time and the dependent variable is distance. The domain and range are all real numbers greater than or equal to zero.

C. $0.50; let s = the cost of each spiral notebook, let b = the cost of each binder purchased, $8s + 4b = 14$

Learn It Online
State Resources Online

Organizer

Objective: Provide review and practice for this chapter and standardized tests.

Online Edition

Resources

Assessment Resources

Chapter Cumulative Test

Learn It Online
State Test Practice

Learn It Online
State Resources Online

CUMULATIVE ASSESSMENT

Multiple Choice

1. Find the value of $|a| - b^2$ when $a = -3$ and $b = -5$.

Ⓐ -28 Ⓒ -7

Ⓑ -22 Ⓓ 4

2. Benito has x apples. He cuts each apple in half and gives each half to a different horse. Which expression represents the number of horses Benito feeds?

Ⓕ $x \cdot \frac{1}{2}$ Ⓗ $x \cdot 1\frac{1}{2}$

Ⓖ $x \div \frac{1}{2}$ Ⓙ $x \div 1\frac{1}{2}$

3. If the value of a^5 is positive, then which is true?

Ⓐ a is positive.

Ⓑ a is negative.

Ⓒ a^5 is odd.

Ⓓ a^5 is even.

4. Find the value of $\frac{2a}{a^3}$ if $4 - a = -6$.

Ⓕ $\frac{1}{50}$ Ⓗ 8

Ⓖ $\frac{1}{2}$ Ⓙ 10

5. There are 36 flowers in a bouquet. Two-thirds of the flowers are roses. One-fourth of the roses are red. What percent of the bouquet is made up of red roses?

Ⓐ 9% Ⓒ 25%

Ⓑ $16\frac{2}{3}\%$ Ⓓ $66\frac{2}{3}\%$

6. A large tree should be planted at least 70 feet away from a power line. Which inequality shows the acceptable number of feet x between a large tree and a power line?

Ⓕ $x < 70$ Ⓗ $x > 70$

Ⓖ $x \leq 70$ Ⓙ $x \geq 70$

7. Which statement is modeled by $3f + 2 > -16$?

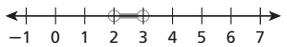
Ⓐ Two added to 3 times f is at least -16.

Ⓑ Three times the sum of f and 2 is at most -16.

Ⓒ The sum of 2 and 3 times f is more than -16.

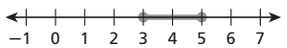
Ⓓ The product of $3f$ and 2 is no more than -16.

8. Jo Ann needs at least 3 pounds of peaches for a recipe. At the market, she calculates that she has enough money to buy 5 pounds at most. Which graph shows all possible numbers of pounds of peaches Jo Ann can buy so that she has enough for the recipe?

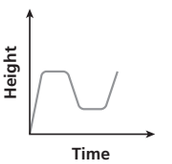
Ⓕ 

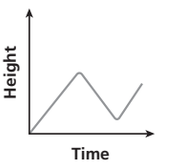
Ⓖ

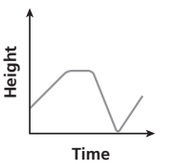
Ⓗ

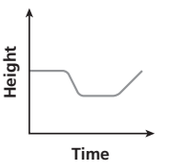
Ⓙ 

9. A bird flies from the ground to the top of a tree, sits there and sings for a while, flies down to the top of a picnic table to eat crumbs, and then flies back to the top of the tree to sing some more. Which graph best represents this situation?

Ⓐ 

Ⓒ 

Ⓑ 

Ⓓ 

10. Which relation is NOT a function?

Ⓕ $\{(1, -5), (3, 1), (-5, 4), (4, -2)\}$

Ⓖ $\{(2, 7), (3, 7), (4, 7), (5, 8)\}$

Ⓗ $\{(1, -5), (-1, 6), (1, 5), (6, -3)\}$

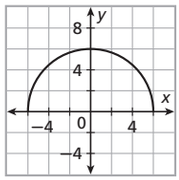
Ⓙ $\{(3, -2), (5, -6), (7, 7), (8, 8)\}$

Answers

1. B
2. G
3. A
4. F
5. B
6. J
7. C
8. J
9. A
10. H
11. D
12. F
13. B
14. 9
15. 72
16. 414

If possible, use the same calculator you usually use in math class. A timed test is not the right place to figure out where buttons are and how they work. Also, replace your batteries the night before the test. If your batteries run out, you may be given a replacement calculator you are not familiar with.

11. The graph below shows a function.



What is the domain of the function?

Ⓐ $x \geq 0$

Ⓑ $x \geq -6$

Ⓒ $0 \leq x \leq 6$

Ⓓ $-6 \leq x \leq 6$

12. Which situation best describes a negative correlation?

Ⓕ The speed of a runner and the time it takes to run a race

Ⓖ The number of apples in a bag and the weight of the bag of apples

Ⓗ The time it takes to repair a car and the amount of the bill

Ⓙ The number of people in a household and the amount of mail in their mailbox

13. Which of the following is a solution of $x + 1 \leq \frac{3}{2}$ AND $x - 1 \geq -\frac{5}{4}$?

Ⓐ $\frac{3}{2}$ Ⓒ $-\frac{1}{3}$

Ⓑ $\frac{1}{3}$ Ⓓ $-\frac{3}{2}$

Gridded Response

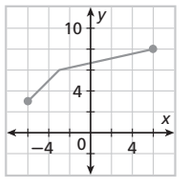
14. What is the value of x when $3(x + 7) - 6x = 4 - (x + 1)$?

15. For $h(x) = x^3 + 2x$, find $h(4)$.

16. WalkieTalkie phone company charges \$18.00 for basic phone service per month and \$0.15 per minute for long distance calls. Arena Calls charges \$80.00 per month with no fee for long distance calls. What is the minimum number of minutes of long distance calls for which the cost of WalkieTalkie is more than the cost of Arena Calls?

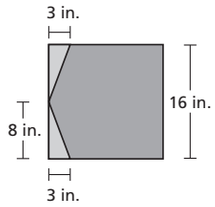
Short Response

17. A function is graphed below.



What is the domain and range of the function?

18. Rory made a pentagon by cutting two triangles from a square piece of cardboard as shown.



What is the area of the pentagon? Show your work or explain how you got your answer.

19. The manager of a new restaurant needs at most 12 servers. He has already hired 7 servers.

a. Write and solve an inequality to determine how many more servers the manager could hire.

b. Graph the solutions to the inequality you solved in part **a.**

20. Study the sequence below.

18, 24.5, 31, 37.5, 44,…

a. Could this sequence be arithmetic? Explain.

b. Find the 100th term of the sequence. Show your work.

Extended Response

21. A relation is shown in the table.

x	y
2	12
3	15
3	18
5	40
6	64

a. Express the relation as a mapping diagram.

b. Is the relation a function? Explain why or why not.

c. Write a possible real-life situation for the relation.

Short-Response Rubric

Items 17–20

2 Points = The student's answer is an accurate and complete execution of the task or tasks.

1 Point = The student's answer contains attributes of an appropriate response but is flawed.

0 Points = The student's answer contains no attributes of an appropriate response.

Extended-Response Rubric

Item 21

4 Points = The student draws the correct mapping diagram in part **a,** gives the correct answer and explanation in part **b,** and writes a possible real-world situation in part **c.**

3 Points = The student draws the correct mapping diagram in part **a,** gives the correct answer but not the correct explanation in part **b,** and writes a possible real-world situation in part **c.**

2 Points = The student draws the correct mapping diagram in part **a,** gives the correct answer and an incorrect explanation in part **b,** and writes an incorrect real-world situation in part **c;** or the student attempts to answer all parts and correctly answers one part.

1 Point = The student draws the mapping diagram with a minor error in part **a,** gives the correct answer but no explanation in part **b,** and attempts to write a possible real-world situation in part **c;** or the student attempts to answer all parts of the problem but does not correctly answer any part.

0 Points = The student does not answer correctly and does not attempt all parts of the problem.

Answers

17. D: $-6 \leq x \leq 6$
R: $3 \leq y \leq 8$

18. 232 in^2

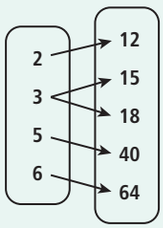
19a. Let s = the number of servers that the manager will hire.
$7 + s \leq 12$
$s \leq 5$ where s is a whole number

b.

20a. Yes; there is a common difference of 6.5.

b. 661.5
$a_{100} = 18 + 99(6.5)$
$a_{100} = 661.5$

21a.



b. No; 3 is mapped to two different values.

c. Possible answer: There were 5 different field trips. The x-values are the number of chaperones, and the y-values are the number of students on each trip.

Real-World CONNECTIONS

Organizer

Objective: Choose appropriate problem-solving strategies and use them to solve real-world problems.

Online Edition

The World's Largest Weathervane

Reading Strategies

Remind students that they should read a word problem at least two times, once to learn the general idea and again to obtain specific information.

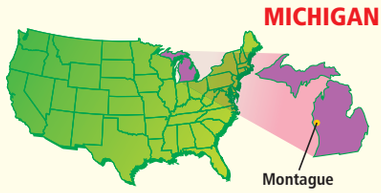
Using Data Have students use the bar graph to make comparison statements such as "The amount of snowfall in January is about four times the amount in March."

Problem-Solving Focus

Encourage students to use the four-step problem-solving process for **Problems 1–2.** Focus on the second step: **(2) Make a Plan.** Ask students what equation the second sentence in **Problem 1** translates into. $h = 2\ell - 4$ Then ask how they can use this equation to solve the problem. Substitute 26 for ℓ. **Problem 2** can be solved similarly, but with an inequality.

Learn It Online
State Resources Online

Real-World CONNECTIONS



The World's Largest Weather Vane

The world's largest working weather vane weighs approximately 4300 pounds and is located in Montague, Michigan. Weather vanes are used to indicate the direction of the wind. When the wind blows, the vane points in the direction from which the wind is coming.

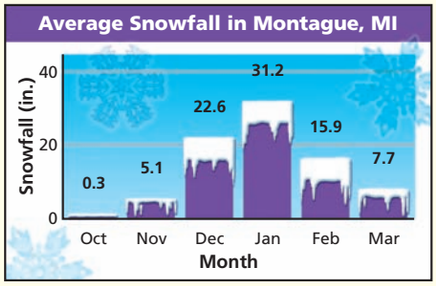
Choose one or more strategies to solve each problem.

1. The arrow on the world's largest weather vane has a length of 26 feet. The height of the weather vane is 4 feet shorter than twice its length. What is the height of the world's largest weather vane? **48 ft**

At the base of the weather vane is a working weather station. Weather stations include instruments such as thermometers, rain gauges, and wind gauges that measure different characteristics of the weather. The data gathered from weather stations are used to make predictions about future weather.

For 2 and 3, use the graph.

2. The data in the graph represent the average snowfall for each month, measured over a 30-year time period. The average amount of snowfall received in April is less than $\frac{1}{4}$ of the average snowfall in March. What is the greatest possible average snowfall for April? Round to the nearest tenth of an inch. **1.9 in.**



3. Which month do you predict will get the most snowfall next year? Explain your reasoning.

January; the data shows that January has the greatest average snowfall for the past 30 years, so it is most likely that January will have the most snowfall next year.

Jess Merril/Alamy

Maple Syrup

Michigan produces about 90,000 gallons of maple syrup each year. This places the state among the top ten states in U.S. production of maple syrup. Maple syrup is made from the sap of maple trees, but only about 1% of Michigan's maple trees are used in maple syrup production.

Choose one or more strategies to solve each problem.

1. The standard sugar concentration level of maple syrup is 66%. At certain levels above 66%, the product develops into maple cream, soft maple sugar, or hard maple sugar. The sugar concentration never reaches 100%, even in hard maple sugar. What is the range of sugar concentration levels in the various maple products? Show your answer on a number line. $0.66 \leq c < 1$

For 2 and 3, use the table.

Sweetener	Calories (per tbsp)
Blackstrap molasses	43
Granulated sugar	46
Maple syrup	50
Corn syrup	57
Honey	64

2. How many Calories are in 1 cup of maple syrup? (*Hint*: 4 tbsp = $\frac{1}{4}$ c) 800 Calories
3. Approximately how many tablespoons of maple syrup would you need to have the same number of Calories that are in 7 tablespoons of honey? Round to the nearest tablespoon. 9 tbsp
4. It takes 40 gallons of maple sap to make 1 gallon of maple syrup. Each tap hole in a maple tree will produce about 10 gallons of sap in an average year. How many gallons of maple syrup could be made with the sap from 20 tap holes? 5 gal
5. It is recommended that maple trees be at least 10 inches in diameter before they are tapped. Only one tap should be placed in trees that are 10 to 18 inches in diameter, while 2 taps can be placed in trees greater than 18 inches in diameter. An orchard has 130 trees that are less than 10 inches in diameter, 104 trees that are 10–18 inches in diameter, and 48 trees that are greater than 18 inches in diameter. What is the maximum number of tap holes this orchard should have? 200

(br), Ed Reschke/Peter Arnold, Inc./Alamy; (bl), © Randy M. Ury/CORBIS; (tc), Sam Dudgeon/HMH

Maple Syrup

Reading Strategies

ENGLISH LANGUAGE LEARNERS

Tell students to underline key words when reading a problem. In **Problem 1,** for example, *never reaches* are key words because they will be used to decide whether an open or closed circle should be used on the number line.

Using Data Ask students how the information in the table is arranged. from least to greatest Calories Have students find the range of Calories for the sweeteners listed in the table. 64 − 43 = 21

Problem-Solving Focus

The information in **Problem 5** can be organized into a table. Ask students what the three categories of trees at the orchard are. less than 10 inches, 10–18 inches, and greater than 18 inches in diameter Then ask them how many trees are in each category. 130, 104, and 48 Then ask them how many taps are in each type of tree. 0, 1, and 2 Have students put the information into a table.

Category	<10	10–18	>18
Number of Trees	130	104	48
Number of Taps per Tree	0	1	2

Ask students how to use the table to answer the question. Multiply the numbers in the second and third rows. Then add those products.